Unit 9 · Transformation

HARDER WORKSHEET · ANSWER KEY

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **b.** a rotation of 180° about O — U · 1

The lines cross at O at right angles, so the two reflections are a rotation about O through $2 \times 90° = 180°$ (and with perpendicular mirrors the order doesn't matter).

ii. **b.** only the origin — A · 1

$R \circ E$ sends (x, y) to $(2x, -2y)$. It stays put only if $2x = x$ and $-2y = y$, so $x = 0$ and $y = 0$: only O. A point on the X-axis is fixed by R, but E moves it.

iii. **b.** -6 — A · 1

The size of the determinant is the area of the image of the unit square, 6. A matrix that flips a figure over has a negative determinant (a reflection's is -1), so it is -6.

iv. **b.** $-\frac{1}{3}$ — A · 1

Two enlargements about one center make one with the product of their scale factors, $3k$. A half turn about the center is the enlargement with scale factor -1, so $3k = -1$ and $k = -\frac{1}{3}$.

2 Short answers

a. MARKING SCHEME — A · 2

- ✓ The rule of $T \circ R$, $(x, y) \to (-y + 2, -x - 1)$, or the two steps undone in reverse order.
- ✓ $P(-4, -3)$.

b. ANSWER — HA · 3

Translate: $(x, y) \to (x + 6, y)$. Reflect in $x = 1$: $(x + 6, y) \to (2 - (x + 6), y) = (-4 - x, y)$. The reflection in $x = h$ is $(x, y) \to (2h - x, y)$; here $2h = -4$, so it is the **reflection in the line $x = -2$**. Check: $(-2, 5)$ is on $x = -2$, so it must stay put: $(-2, 5) \to (4, 5) \to (2 - 4, 5) = (-2, 5)$. ✓

MARKING SCHEME

- ✓ The combined rule $(x, y) \to (-4 - x, y)$ worked out.
- ✓ Compared with $(2h - x, y)$: the reflection in $x = -2$.
- ✓ The check with $(-2, 5)$, which stays where it is.

c. ANSWER — HA · 3

$R \circ T$: T first, $(x, y) \to (x, y + 4)$, then R, $\to (y + 4, x)$. A point stays put if $y + 4 = x$ and $x = y$; then $y + 4 = y$, which is impossible. **No point stays where it is.** A reflection keeps every point of its mirror line, and a rotation keeps its center, so $R \circ T$ is neither. (It is a reflection in the line $y = x - 2$ followed by a slide of $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ along that line, a **glide reflection**, like a trail of footprints.)

MARKING SCHEME

- ✓ The rule $(x, y) \to (y + 4, x)$, T done first.
- ✓ The equations $y + 4 = x$ and $x = y$ have no solution: no fixed point.
- ✓ So it is not a reflection (no mirror line kept) and not a rotation (no center kept).

d. MARKING SCHEME — A · 3

- ✓ $a = 1$ and $b = 3$.
- ✓ $\det M = -3$, so the area is $3 \times 4 = 12$ square units.
- ✓ Negative determinant: the image is flipped over.

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e. MARKING SCHEME A · 2

✓ $M^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$ found.

✓ $P(3, 1)$.

f. MARKING SCHEME U · 2

✓ The order is wrong: "first A, then B" is BA, not AB.

✓ $BA = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the reflection in $y = x$.

3 Long answers

a. ANSWER A · 4

i. E first: $(x, y) \to (-2(x - 1) + 1, -2y) = (3 - 2x, -2y)$: $K'(-1, -2)$, $L'(-5, -2)$, $M'(-3, -6)$. Then R, $(x, y) \to (y, x)$: K''**(−2, −1),** L''**(−2, −5),** M''**(−6, −3)**.

ii. On graph paper, x and y from -6 to 4: $\triangle KLM$ in the first quadrant and $\triangle K''L''M''$ in the third.

iii. $\triangle KLM$ has base $KL = 2$ and height 2, area 2. The enlargement multiplies areas by $(-2)^2 = 4$, and a reflection keeps areas, so $\triangle K''L''M''$ has area **8**, four times as much. E with a negative scale factor turns the triangle round (a half turn and a stretch) but doesn't flip it; the reflection then flips it, so the final image is a **mirror image** of $\triangle KLM$.

MARKING SCHEME

✓ E's images $K'(-1, -2)$, $L'(-5, -2)$, $M'(-3, -6)$.

✓ R's images $K''(-2, -1)$, $L''(-2, -5)$, $M''(-6, -3)$.

✓ Both triangles drawn and labeled on one graph with a scale.

✓ Area $8 = 4 \times 2$, and the image is a mirror image.

b. ANSWER A · 4

i. Turn 90° clockwise $R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; flip left to right (in the Y-axis) $F = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; turn 180° $H = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

ii. R first, then F, then H: $FR = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, and $H(FR) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

iii. The **reflection in the line** $y = x$, a flip in one diagonal. Its determinant is -1, so the photo shown is a mirror image. (Each turn keeps the picture the right way round; the one flip makes it mirrored, however many turns there are.)

MARKING SCHEME

✓ The three matrices R, F and H.

✓ FR $= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, in the right order.

✓ $H(FR) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

✓ The reflection in $y = x$: a mirror image.

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c. **ANSWER** HA · 3

Two of many: the reflection in $y = x$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, and $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{pmatrix}$ (half the x, twice the y); check: $(2, 1) \to (1, 2)$ for both. With B too: the image of $(1, 0)$ is the first column, so it is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$; then $A = 2 \times (1, 0) + (0, 1)$ goes to $2 \times (0, 1) +$ (second column) $= (1, 2)$, so the second column is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. There is **exactly one**, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$: two points not in line with O fix a matrix completely, which is why the image of the unit square gives the whole matrix.

MARKING SCHEME

- ✓ Two different correct matrices, each checked on A.
- ✓ With B as well, the first column is $(0, 1)$ and the second $(1, 0)$.
- ✓ Exactly one such matrix, the reflection in $y = x$, with the reason (two points not in line with O fix the matrix).

4 Think

ANSWER HA · 2

Each pair of reflections is a turn of $2 \times 36° = \mathbf{72}°$, and $360° \div 36° = \mathbf{10}$ copies fill the circle (five turned copies and five flipped ones). 45°, 36° and 30° divide 180° exactly ($180° \div n$), so after n turns the copies come back exactly to the start and meet edge to edge. At 50°, $360° \div 50°$ is not a whole number, so the copies overlap unevenly and the pattern doesn't close into a neat circle.

MARKING SCHEME

- ✓ A turn of 72° and 10 copies.
- ✓ The angle must divide 180° (or 360° into an even number of pieces) so the copies meet exactly; 50° doesn't.

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- **The Kaleidoscope** (exhibit): Turn the knob to change the angle between the mirrors: two reflections make a turn of twice that angle, so a smaller angle gives more copies round the circle.
- **The Periscope** (exhibit): Two parallel mirrors: read some writing through it. Two reflections in parallel mirrors make a slide, so the writing is not reversed.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	18	11	**32**

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