Unit 9 · Transformation

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. isometric
2. translation vector
3. combined
4. F
5. translation
6. rotation

Each carries 0.5 mark. **K · 3**

2 Match the columns

1. $P'(y, x) \rightarrow$ **g.** reflection in the line $y = x$
2. $P'(-y, -x) \rightarrow$ **a.** reflection in the line $y = -x$
3. $P'(-y, x) \rightarrow$ **f.** rotation of $+90^\circ$ about O
4. $P'(y, -x) \rightarrow$ **c.** rotation of -90° about O
5. $P'(4 - x, y) \rightarrow$ **b.** reflection in the line $x = 2$
6. $P'(-2x, -2y) \rightarrow$ **d.** enlargement $E[(0, 0), -2]$

Each carries 0.5 mark. **K · 3**

3 Multiple choice

i. **c.** a translation **K · 1**

Only a translation moves every point by the same vector. In a reflection or a rotation, points far from the mirror or the center move farther.

ii. **b.** a translation 6 units to the right **U · 1**

Parallel mirrors 3 units apart make a translation through twice the gap, 6 units, from the first line towards the second (to the right).

iii. **a.** a rotation of -90° about O **A · 1**

Two rotations about one center add: $+90^\circ + 180^\circ = +270^\circ$, which ends where -90° does.

iv. **b.** $(0, 0)$ **U · 1**

Any 2×2 matrix times the column of $(0, 0)$ gives $(0, 0)$: every matrix transformation keeps the origin where it is.

v. **b.** a rotation of 180° about O **A · 1**

The scale factors multiply: $3 \times (-\frac{1}{3}) = -1$, and $E[(0, 0), -1]$ sends (x, y) to $(-x, -y)$, the half turn about O.

4 Very short answers

a. **MARKING SCHEME** **K · 1**

✓ $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$ written.

b. **MARKING SCHEME** **U · 1**

✓ Rotation of 180° about O (both the angle and the center needed).

c. **MARKING SCHEME** **A · 1**

✓ $(-5, 3)$ worked out.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

5 Short answers

a. MARKING SCHEME U · 2

- ✓ The order is wrong: in $R \circ T$ the translation T comes first.
- ✓ $P''(-5, 1)$ found.

b. MARKING SCHEME A · 2

- ✓ $3(x - 1) + 1 = 7$ and $3(y - 2) + 2 = -4$ written.
- ✓ $P(3, 0)$.

c. MARKING SCHEME U · 2

- ✓ Two reflections in parallel lines (mirrors) make a translation.
- ✓ A translation doesn't reverse (flip) the picture, so the writing reads the right way.

d. MARKING SCHEME A · 2

- ✓ BA (second matrix on the left) worked out: $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$.
- ✓ Enlargement $E[(0, 0), 2]$ together with a rotation of $+90^\circ$ about O.

6 Long answers

a. ANSWER A · 4

i. In $x = -1$, $P(x, y) \to P'(-2 - x, y)$: $A'(-3, 1)$, $B'(-6, 1)$, $C'(-3, 3)$. In $x = 3$, $P'(x, y) \to P''(6 - x, y)$: $A''(9, 1)$, $B''(12, 1)$, $C''(9, 3)$.

ii. The mirrors are parallel, 4 units apart, so it is the translation $2\begin{pmatrix} 3 - (-1) \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$, 8 units to the right. (Check: $A(1, 1) + 8 = A''(9, 1)$.)

iii. On graph paper, x from -6 to 12: $\triangle ABC$, its mirror image $\triangle A'B'C'$ to the left of $x = -1$, and $\triangle A''B''C''$, the same way round as $\triangle ABC$, 8 units to its right.

MARKING SCHEME

- ✓ $A'(-3, 1)$, $B'(-6, 1)$, $C'(-3, 3)$.
- ✓ $A''(9, 1)$, $B''(12, 1)$, $C''(9, 3)$.
- ✓ The translation $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$ (8 units to the right).
- ✓ The graph: axes with a scale, the three triangles labeled, both mirror lines.

b. ANSWER A · 4

i. The first column is A', the second C': $M = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$.

ii. The area is $|\det M| = |2 \times 3 - 1 \times 1| =$ **5 square units** (the unit square's area 1 multiplied by 5).

iii. $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$, so **(0, 5)**.

iv. **None of them.** A reflection or a rotation keeps the area (determinant ± 1), and an enlargement keeps the shape, so it would turn the square into a square. M turns the square into a parallelogram of area 5.

MARKING SCHEME

- ✓ $M = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$.
- ✓ Area $= |\det M| = 5$ square units.
- ✓ Image (0, 5).
- ✓ Not one of them, with a reason (area changes, so not a reflection or rotation; shape changes, so not an enlargement).

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

7 Think

ANSWER HA · 2

Turn 90° clockwise: $R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; flip top to bottom (in the X-axis): $F = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. R comes first, so the single matrix is $FR = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the reflection in the line $y = x$. **She is wrong**: the photo shown is a mirror image, flipped in a diagonal, and no turn can make a mirror image (a turn's determinant is 1, this one's −1).

MARKING SCHEME

✓ $FR = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

✓ Not a turn but the reflection in $y = x$: the photo is mirrored, so Gita is wrong.

8 Activity: Two mirrors make a turn

(not marked)

WHAT STUDENTS SHOULD FIND

Angle	Turn (2 × the angle)	Images	Coins in all
120°	240°	2	3
90°	180°	3	4
60°	120°	5	6
45°	90°	7	8
The coins in all are 360° ÷ the angle, so the images are one fewer. Each pair of reflections turns the picture through twice the angle between the mirrors, so the copies go round the circle; this is how a kaleidoscope works.			

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **The Kaleidoscope** (exhibit): Turn the knob to change the angle between the mirrors: two reflections make a turn of twice that angle, so a smaller angle gives more copies round the circle.
- **The Periscope** (exhibit): Two parallel mirrors: read some writing through it. Two reflections in parallel mirrors make a slide, so the writing is not reversed.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	8	7	15	2	**32**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking