Unit 8 · Coordinate Geometry

HARDER WORKSHEET · ANSWER KEY

Full marks 24

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **c.** $k = \frac{2}{3}$ U · 1

The slopes are $-\frac{k}{2}$ and 3. Perpendicular: $-\frac{3k}{2} = -1$, so $k = \frac{2}{3}$. ($k = -6$ would make them parallel.)

ii. **a.** $x^2 + y^2 - 6x - 6y + 9 = 0$ A · 1

The first has center (3, 3) and radius $\sqrt{9 + 9 - 9} = 3$: 3 from each axis. The second passes through the origin (radius $\sqrt{18}$); the third has center (3, 2) and radius 2, touching only the X-axis; the fourth has radius $\sqrt{15}$.

iii. **c.** $\alpha > 20°$ HA · 1

Two branches make a hyperbola, which needs $\theta < \alpha$: so α is more than 20°.

2 Short answers

a. MARKING SCHEME A · 2

- ✓ $k^2 - 4k - 5 = 0$ from substituting.
- ✓ $k = 5$ or $k = -1$.

b. MARKING SCHEME A · 2

- ✓ The tangent's slope $-\frac{3}{4}$, perpendicular to the radius.
- ✓ $3x + 4y = 25$.

c. ANSWER HA · 3

The first: center (0, 0), radius 5. The second: center (12, 9), radius $\sqrt{144 + 81 - 125} = 10$. The distance between the centers is $\sqrt{144 + 81} = 15 = 5 + 10$, the sum of the radii, so they touch (from outside).

The point is 5 along the line of centers from O, that is $\frac{5}{15}$ of the way to (12, 9): **(4, 3)**. Check:

$16 + 9 = 25$, and $16 + 9 - 96 - 54 + 125 = 0$.

MARKING SCHEME

- ✓ Both centers and radii: (0, 0), 5 and (12, 9), 10.
- ✓ Distance between centers $15 = 5 + 10$, so they touch.
- ✓ The point (4, 3).

3 Long answers

a. ANSWER A · 4

i. Subtracting: $4x + 2y = 10$, that is $\mathbf{2x + y = 5}$.

ii. $y = 5 - 2x$ in $x^2 + y^2 = 10$: $5x^2 - 20x + 15 = 0$, $x^2 - 4x + 3 = 0$, $x = 1$ or 3. **(1, 3) and (3, −1)**.

iii. $\sqrt{(3-1)^2 + (-1-3)^2} = \sqrt{20} = 2\sqrt{5}$ units.

MARKING SCHEME

- ✓ $2x + y = 5$.
- ✓ The quadratic $x^2 - 4x + 3 = 0$ formed.
- ✓ (1, 3) and (3, −1).
- ✓ $2\sqrt{5}$ units.

b. ANSWER

i. AB is the X-axis, so the altitude from C is upright: $x = 2$. BC has slope $\frac{4}{-4} = -1$, so the altitude from A has slope 1: $y = x$.

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ii. $H(\mathbf{2, 2})$. **A · 4**

iii. AC has slope 2, so the altitude from B has slope $-\frac{1}{2}$: $y = -\frac{1}{2}(x - 6)$, that is $x + 2y - 6 = 0$. At H: $2 + 4 - 6 = 0$. ✓

MARKING SCHEME

- ✓ Altitude from C: $x = 2$.
- ✓ Altitude from A: $y = x$.
- ✓ $H(2, 2)$.
- ✓ Altitude from B, $x + 2y - 6 = 0$, checked through H.

c. **ANSWER** **A · 4**

i. Slopes $\frac{1}{2}$ and 3: $\dfrac{3 - \frac{1}{2}}{1 + \frac{3}{2}} = 1$, so the angle is **45**°.

ii. The second road is $y = 3x$; the path has slope $-\frac{1}{3}$ through P: $3y - 6 = -x + 4$, $\mathbf{x + 3y - 10 = 0}$.

iii. With $y = 3x$: $10x = 10$, so it meets the road at **(1, 3)**, which is Q. Length $\sqrt{9 + 1} = \sqrt{10} \approx 3.16$ units, about **316 m**.

MARKING SCHEME

- ✓ The angle 45°.
- ✓ The path $x + 3y - 10 = 0$.
- ✓ The foot (1, 3).
- ✓ The length $\sqrt{10}$ units, about 316 m.

4 Think

ANSWER **HA · 2**

No. Its "center" would be $(-2, 3)$, but $r^2 = g^2 + f^2 - c = 4 + 9 - 20 = -7$, and no radius squared is negative. Written as $(x + 2)^2 + (y - 3)^2 = -7$, the left side is never negative, so **no point** satisfies it: it is not a circle. The equation is a circle only when $g^2 + f^2 - c > 0$.

MARKING SCHEME

- ✓ $r^2 = 4 + 9 - 20 = -7$, negative.
- ✓ So no point satisfies it: not a circle (needs $g^2 + f^2 - c > 0$).

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	1	17	6	**24**

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