Unit 7 · Trigonometry

HARDER WORKSHEET · ANSWER KEY

Full marks 23

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **b.** 45° **A · 1**

$\tan(A+B) = \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \dfrac{5/6}{5/6} = 1$, so $A + B = 45°$.

ii. **c.** $\frac{1}{2}$ **U · 1**

$\sin x \cos x = \frac{1}{2}\sin 2x$, and $\sin 2x$ is at most 1 (at $x = 45°$).

iii. **d.** $\frac{1}{8}$ **A · 1**

Multiply by $2\sin 20°$ three times: $2\sin 20° \cos 20° = \sin 40°$, then $\sin 80°$, then $\sin 160° = \sin 20°$. So $8\sin 20° \times$ the product $= \sin 20°$, and the product is $\frac{1}{8}$.

2 Short answers

a. **MARKING SCHEME** **U · 2**

- ✓ $\sin 2A = 2\sin A \cos A$, not $2\sin A$.
- ✓ $\sin 60° = \dfrac{\sqrt{3}}{2}$.

b. **MARKING SCHEME** **A · 2**

- ✓ $\sin A = \frac{4}{5}$ and $\cos A = \frac{3}{5}$.
- ✓ $\tan A = \frac{4}{3}$.

c. **ANSWER** **HA · 2**

$\sin(60° - A)\sin(60° + A) = \sin^2 60° - \sin^2 A = \frac{3}{4} - \sin^2 A$. So the right side is

$4\sin A(\frac{3}{4} - \sin^2 A) = 3\sin A - 4\sin^3 A = \sin 3A$.

MARKING SCHEME

- ✓ $\sin(60° - A)\sin(60° + A) = \dfrac{3}{4} - \sin^2 A$.
- ✓ $3\sin A - 4\sin^3 A = \sin 3A$.

d. **ANSWER** **HA · 2**

No. The top of the flag is higher than the top of the tower, and from the same point a higher point is seen at a bigger angle of elevation. So the flag's angle must be more than 60°, not 45°. (The tower would be $30\sqrt{3} \approx 52$ m high but the flag's top only 30 m.)

MARKING SCHEME

- ✓ Not possible.
- ✓ A higher point on the same upright line has the bigger angle, so the flag's must be more than 60°.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

3 Long answers

a. **ANSWER** **A · 4**

Let the hill be h m high and the point d m away.

The top at 45°: $h + 40 = d \tan 45° = d$. The foot at 30°: $h = d \tan 30° = \frac{d}{\sqrt{3}}$.

So $d = \frac{d}{\sqrt{3}} + 40$, $d\left(1 - \frac{1}{\sqrt{3}}\right) = 40$, $d = \frac{40\sqrt{3}}{\sqrt{3} - 1} = 20(3 + \sqrt{3}) \approx$ **94.64 m**.

Hill $h = d - 40 \approx$ **54.64 m**.

MARKING SCHEME

- ✓ $h + 40 = d$.
- ✓ $h = d/\sqrt{3}$.
- ✓ $d = 20(3 + \sqrt{3}) \approx 94.64$ m.
- ✓ $h \approx 54.64$ m.

b. **ANSWER** **HA · 4**

$\cos^2 A + \cos^2 B = 1 + \frac{1}{2}(\cos 2A + \cos 2B) = 1 + \cos(A + B)\cos(A - B) = 1 - \cos C \cos(A - B)$, since $\cos(A + B) = -\cos C$.

Adding $\cos^2 C = -\cos C \cos(A + B)$:

$1 - \cos C[\cos(A - B) + \cos(A + B)] = 1 - \cos C \cdot 2\cos A \cos B = 1 - 2\cos A \cos B \cos C$.

MARKING SCHEME

- ✓ $\cos^2 A + \cos^2 B = 1 + \cos(A + B)\cos(A - B)$.
- ✓ $\cos(A + B) = -\cos C$, so $\cos^2 C = -\cos C \cos(A + B)$.
- ✓ $1 - \cos C[\cos(A - B) + \cos(A + B)]$.
- ✓ $= 1 - 2\cos A \cos B \cos C$.

4 Think

a. **ANSWER** **HA · 2**

They are equal. $\sin 40° + \sin 20° = 2\sin 30° \cos 10° = \cos 10°$, and $\cos 10° = \sin 80°$.

MARKING SCHEME

- ✓ Sum to product: $2\sin 30° \cos 10° = \cos 10°$.
- ✓ $\cos 10° = \sin 80°$: they are equal.

b. **ANSWER** **HA · 2**

Any pair with $\tan A + \tan B = 1 - \tan A \tan B$ works. For example $\tan A = \frac{1}{4}$, $\tan B = \frac{3}{5}$:

$\frac{1}{4} + \frac{3}{5} = \frac{17}{20}$ and $1 - \frac{3}{20} = \frac{17}{20}$, so $\tan(A + B) = 1$ and $A + B = 45°$. (Others: $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{5}$ and $\frac{2}{3}$.)

MARKING SCHEME

- ✓ A pair with $\tan A + \tan B = 1 - \tan A \tan B$.
- ✓ Shown to give $\tan(A + B) = 1$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	8	12	**23**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking