Unit 7 · Trigonometry

ANSWER KEY · FOR TEACHERS

Full marks 30

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. compound angle
2. multiple angles
3. submultiple angle
4. line of sight
5. angle of elevation
6. angle of depression

K · 3

Each carries 0.5 mark.

2 Read the figure

1. 45° 2. 60°

U · 1

Each carries 0.5 mark.

3 Multiple choice

i. **a.** $\sin A \cos B + \cos A \sin B$ K · 1

This is the compound angle formula for the sine. $\sin A + \sin B$ is the tempting wrong answer: $\sin(30° + 60°) = 1$, but $\sin 30° + \sin 60° \approx 1.37$.

ii. **c.** $1 - 2\sin^2 A$ K · 1

$\cos 2A = \cos^2 A - \sin^2 A = (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$. The last option is always 1.

iii. **b.** $\dfrac{\sqrt{3} - 1}{2\sqrt{2}}$ A · 1

$\sin(45° - 30°) = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2} = \dfrac{\sqrt{3} - 1}{2\sqrt{2}} \approx 0.259$. The first option is $\sin 75°$.

iv. **c.** $2\sin 4A \cos A$ U · 1

$\sin C + \sin D = 2\sin \dfrac{C + D}{2} \cos \dfrac{C - D}{2}$: half of $8A$ is $4A$, half of $2A$ is A.

v. **d.** $30°$ A · 1

$\tan\theta = \dfrac{20}{20\sqrt{3}} = \dfrac{1}{\sqrt{3}}$, so $\theta = 30°$. A shadow longer than the pole means the sun is lower than 45°.

4 Very short answers

a. MARKING SCHEME K · 1

✓ $\dfrac{2\tan A}{1 - \tan^2 A}$.

b. MARKING SCHEME U · 1

✓ $\sin 50° + \sin 30°$.

c. MARKING SCHEME A · 1

✓ 2.

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5 Short answers

a. MARKING SCHEME U · 2

- ✓ $\cos(A+B) \neq \cos A + \cos B$ (a cosine can't be more than 1).
- ✓ $\cos 75^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$.

b. MARKING SCHEME U · 2

- ✓ $\sin 2A = 2\sin A \cos A$ and $1 + \cos 2A = 2\cos^2 A$.
- ✓ Cancelling to $\tan A$.

c. MARKING SCHEME A · 2

- ✓ $\sin A = \dfrac{4}{5}$ and $\sin 2A = \dfrac{24}{25}$.
- ✓ $\cos 2A = -\dfrac{7}{25}$.

d. MARKING SCHEME A · 2

- ✓ Distances 30 ft and $10\sqrt{3}$ ft from the foot.
- ✓ 12.68 ft apart.

6 Long answers

a. ANSWER A · 4

$\sin 2A + \sin 2B = 2\sin(A+B)\cos(A-B) = 2\sin C \cos(A-B)$, since $A + B = \pi - C$.
$\sin 2C = 2\sin C \cos C = -2\sin C \cos(A+B)$, since $\cos C = \cos(\pi - (A+B)) = -\cos(A+B)$.
Adding: $2\sin C[\cos(A-B) - \cos(A+B)] = 2\sin C \cdot 2\sin A \sin B = 4\sin A \sin B \sin C$.

MARKING SCHEME

- ✓ $\sin 2A + \sin 2B = 2\sin(A+B)\cos(A-B)$.
- ✓ $\sin(A+B) = \sin C$ and $\cos C = -\cos(A+B)$.
- ✓ $2\sin C[\cos(A-B) - \cos(A+B)]$.
- ✓ $= 4\sin A \sin B \sin C$.

b. ANSWER A · 4

Let the building be AB (A the roof) and the column CD (C its top), d m apart.
The foot D at 60°: $\tan 60^\circ = \dfrac{30}{d}$, so $d = \dfrac{30}{\sqrt{3}} = 10\sqrt{3} \approx 17.32$ m.
The top C at 45°: it is $d \tan 45^\circ = 17.32$ m below the roof.
Height of the column $= 30 - 17.32 =$ **12.68 m**.

MARKING SCHEME

- ✓ A figure with both angles of depression marked.
- ✓ $d = 10\sqrt{3} \approx 17.32$ m.
- ✓ The top is 17.32 m below the roof.
- ✓ Column 12.68 m.

7 Think

ANSWER HA · 2

No. Doubling the distance halves $\tan\theta$, not the angle. For a tree 10 m high: from 10 m, $\tan\theta = 1$, $\theta = 45^\circ$; from 20 m, $\tan\theta = \dfrac{1}{2}$, about 26.6°, not 22.5°.

MARKING SCHEME

- ✓ No: the tangent halves, not the angle.
- ✓ An example worked, e.g. 45° and about 26.6°.

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8 Activity: How tall is your school?

(not marked)

WHAT STUDENTS SHOULD FIND

The three heights agree to within about half a meter if the angles are read carefully. For example, 10 m away at 40° with the eye 1.5 m up gives $10 \times 0.839 + 1.5 \approx 9.9$ m. A one-degree slip in the angle changes the answer least when the angle is near 45° (standing about as far away as the building is tall); very steep or very shallow angles give the worst answers. Without the eye height, every answer is too low by the same amount.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	6	7	15	2	**30**

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