Unit 6 · Matrix and Determinant

WORKSHEET

Full marks 32

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 32

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

determinant	**identity**	**inverse**	**matrix**	**rectangle**	**singular**	**symmetric**	**transpose**

Numbers arranged in rows and columns, in brackets, make a _____. Writing its rows as columns gives its _____. A square matrix that equals its own transpose is called _____. For $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the number $ad - bc$ is its _____. A matrix whose determinant is 0 is _____, and it has no _____.

2 Multiple choice [5]

Choose the correct option for each of the following questions.

i. What is meant by the transpose of a matrix? [1]

a. changing its rows only
b. changing its columns only
c. changing its signs only
d. interchanging its rows and columns

ii. When can two matrices be multiplied? [1]

a. when the number of columns of the first equals the number of rows of the second
b. when the number of rows of the first equals the number of columns of the second
c. when the number of rows of the first equals the number of rows of the second
d. when the number of columns of the first equals the number of columns of the second

iii. When is the inverse A^{-1} of a matrix A not defined? [1]

a. when A is a singular matrix
b. when A is a zero matrix only
c. when A is a square matrix
d. when A is an identity matrix

iv. What is the determinant of $\begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}$? [1]

a. -5
b. 5
c. 11
d. -11

v. A is a 2×3 matrix and B is a 3×4 matrix. Which of these can be found? [1]

a. AB, of order 2×4
b. BA, of order 3×3
c. AB, of order 3×3
d. neither AB nor BA

3 Very short answers [3]

Write very short answers to the following questions.

a. If $A = \begin{pmatrix} 2 & -1 & 0 \\ 5 & 3 & 7 \end{pmatrix}$, write its transpose A^{T} (also written A'). [1]

b. Find the determinant of $\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$. Is the matrix singular? **[1]**

c. Find $\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix}$. **[1]**

4 Short answers **[8]**

Write short answers to the following questions.

a. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$, find AB and BA. Are they equal? **[2]**

b. For what value of x is $\begin{pmatrix} x & 6 \\ 2 & 4 \end{pmatrix}$ a singular matrix? **[2]**

c. Find the mistake. Asha finds the inverse of $A = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}$: "$|A| = 1$, so $A^{-1} = \begin{pmatrix} 1 & -1 \\ -3 & 4 \end{pmatrix}$." What has she done wrong? Write the correct inverse. **[2]**

d. Work backwards. The inverse of a matrix A is $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$. Find A. **[2]**

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5 Long answers [11]

Solve the following problems. Show your working.

a. Prove each of these. [3]

a. $A^2 = 2A$, where $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$.

b. XY is a zero matrix, where $X = \begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}$ and $Y = \begin{pmatrix} 2 & -5 \\ 2 & -5 \end{pmatrix}$.

c. $R^2 = I$, where $R = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$.

b. $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$. [4]

a. Find $|A|$ and A^{-1}.

b. Find AB and $(AB)^{-1}$.

c. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Answer:

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c. $P = \begin{pmatrix} 2m & 7 \\ 5 & 9 \end{pmatrix}$, and its inverse is $Q = \begin{pmatrix} 9 & n \\ -5 & 4 \end{pmatrix}$. **[4]**

a. Write the determinant of an identity matrix.

b. Find m and n.

c. Find the determinant of P.

d. Find Q^{T}.

6 Think **[2]**

Think, then answer.

Ram says: "The matrix $\begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$ would be a bad key for a secret code." Is he right? Explain with its determinant.

7 Activity: Send a word in code (not marked)

Work with a partner. Turn a 4-letter word into numbers (A = 1, B = 2, … Z = 26), take them two at a time as columns, and multiply each by the key $K = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$. Give your partner only the coded numbers. Your partner finds K^{-1} and reads your word back.

letters	numbers	coded: K × column	read back: K^{-1} × coded