Unit 6 · Matrix and Determinant

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. matrix
2. transpose
3. symmetric
4. determinant
5. singular
6. inverse

K · 3

Each carries 0.5 mark.

2 Multiple choice

i. **d.** interchanging its rows and columns — K · 1

The transpose writes every row as a column, so rows and columns are interchanged together. No sign changes.

ii. **a.** when the number of columns of the first equals the number of rows of the second — K · 1

Each row of the first is paired with each column of the second, so they must have the same number of elements.

iii. **a.** when A is a singular matrix — K · 1

A singular matrix, $|A| = 0$, has no inverse. A zero matrix is one of many singular matrices, and $I^{-1} = I$.

iv. **c.** 11 — A · 1

$3 \times 1 - (-2) \times 4 = 3 + 8 = 11$.

v. **a.** AB, of order 2×4 — U · 1

$(2 \times 3)(3 \times 4)$: the inside numbers match, so AB can be found, 2×4. BA is $(3 \times 4)(2 \times 3)$: 4 and 2 don't match.

3 Very short answers

a. MARKING SCHEME — K · 1

✓ The transpose, with rows written as columns (order 3×2).

b. MARKING SCHEME — U · 1

✓ -2, so not singular.

c. MARKING SCHEME — A · 1

✓ (-5).

4 Short answers

a. MARKING SCHEME — U · 2

✓ $AB = \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$.

✓ $BA = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$, so $AB \neq BA$.

b. MARKING SCHEME — A · 2

✓ $4x - 12 = 0$.

✓ $x = 3$.

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c. **MARKING SCHEME** U · 2

✓ The other diagonal's elements are not swapped, only their signs changed.

✓ $A^{-1} = \begin{pmatrix} 1 & -3 \\ -1 & 4 \end{pmatrix}$.

d. **MARKING SCHEME** A · 2

✓ A is the inverse of the given matrix; its determinant is 1.

✓ $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$.

5 Long answers

a. **MARKING SCHEME** U · 3

✓ $A^2 = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} = 2A$.

✓ $XY = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

✓ $R^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$.

b. **ANSWER** A · 4

a. $|A| = 6 - 5 = 1, A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$.

b. $AB = \begin{pmatrix} 3+1 & 6+3 \\ 5+2 & 10+6 \end{pmatrix} = \begin{pmatrix} 4 & 9 \\ 7 & 16 \end{pmatrix}$; $|AB| = 64 - 63 = 1$, so $(AB)^{-1} = \begin{pmatrix} 16 & -9 \\ -7 & 4 \end{pmatrix}$.

c. $|B| = 3 - 2 = 1, B^{-1} = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$, and $B^{-1}A^{-1} = \begin{pmatrix} 6+10 & -3-6 \\ -2-5 & 1+3 \end{pmatrix} = \begin{pmatrix} 16 & -9 \\ -7 & 4 \end{pmatrix}$, the same as b.

MARKING SCHEME

✓ $|A| = 1$ and $A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$.

✓ $AB = \begin{pmatrix} 4 & 9 \\ 7 & 16 \end{pmatrix}$.

✓ $(AB)^{-1} = \begin{pmatrix} 16 & -9 \\ -7 & 4 \end{pmatrix}$.

✓ $B^{-1} = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$ and $B^{-1}A^{-1}$ equal to $(AB)^{-1}$.

c. **ANSWER** A · 4

a. $|I| = 1 \times 1 - 0 \times 0 = \mathbf{1}$.

b. $PQ = I$, as in Example 2 of 6.5: $\boldsymbol{m = 2, n = -7}$.

c. $P = \begin{pmatrix} 4 & 7 \\ 5 & 9 \end{pmatrix}$: $|P| = 36 - 35 = \mathbf{1}$.

d. $Q = \begin{pmatrix} 9 & -7 \\ -5 & 4 \end{pmatrix}$, so $Q^{\mathrm{T}} = \begin{pmatrix} \mathbf{9} & \mathbf{-5} \\ \mathbf{-7} & \mathbf{4} \end{pmatrix}$.

MARKING SCHEME

✓ $|I| = 1$.

✓ $PQ = I$ gives $18m - 35 = 1$ and $5n + 36 = 1$: $m = 2$, $n = -7$.

✓ $|P| = 36 - 35 = 1$.

✓ $Q^{\mathrm{T}} = \begin{pmatrix} 9 & -5 \\ -7 & 4 \end{pmatrix}$.

6 Think

ANSWER HA · 2

Yes. Its determinant is $12 - 12 = 0$, so it has no inverse and the code can't be undone. Different pairs give the same code: (2, 0) and (0, 1) both become (4, 6).

MARKING SCHEME

✓ The determinant is 0, so there is no inverse to decode with.

✓ Two different pairs give the same code, e.g. (2, 0) and (0, 1) both give (4, 6).

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7 Activity: Send a word in code

(not marked)

WHAT STUDENTS SHOULD FIND

$|K| = 3 - 2 = 1$, so $K^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$. For the word GOAL: G, O are 7, 15, coded $(3 \times 7 + 15, 2 \times 7 + 15) = (36, 29)$; A, L are 1, 12, coded (15, 14). Reading back: $K^{-1} \begin{pmatrix} 36 \\ 29 \end{pmatrix} = \begin{pmatrix} 36 - 29 \\ -72 + 87 \end{pmatrix} = \begin{pmatrix} 7 \\ 15 \end{pmatrix}$, G and O again. Every word comes back, because $K^{-1}K = I$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	7	9	14	2	**32**

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