Unit 5 · Surd

HARDER WORKSHEET · ANSWER KEY

Full marks 21

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **d.** none — U · 1

A square root is never negative, so no x works. Squaring gives $x = 9$, but $\sqrt{4} = 2 \neq -2$: a false root.

ii. **b.** 6 — A · 1

$\sqrt{3+k} = 3$, so $3 + k = 9$ and $k = 6$. (Then $x^2 - x - 6 = 0$ also gives $x = -2$, a false root.)

iii. **b.** $\sqrt[3]{ax+b} = x$ — U · 1

Cubing keeps the sign: $A^3 = B^3$ only when $A = B$. Squaring also accepts $A = -B$.

2 Short answers

a. MARKING SCHEME — A · 2

- ✓ $\sqrt{x} = 4$.
- ✓ $x = 16$, checked.

b. MARKING SCHEME — U · 2

- ✓ The middle term $4\sqrt{x}$ was left out.
- ✓ $x = 1$, checked.

c. ANSWER — HA · 2

Squaring: $x + a = x^2 - 2bx + b^2$, so $x^2 - (2b+1)x + b^2 - a = 0$. Matching, $2b + 1 = 9$, $b = 4$, and $16 - a = 14$, $a = 2$: $\sqrt{x+2} = x - 4$.
Its solutions are 2 and 7. At $x = 2$, $x - 4 = -2$ is negative: **2 is the false root** ($\sqrt{4} = 2 \neq -2$); 7 is true ($\sqrt{9} = 3$).

MARKING SCHEME

- ✓ $a = 2$, $b = 4$.
- ✓ 2 is the false root, with the reason.

d. ANSWER — HA · 2

The left side times $\sqrt{x+4} + \sqrt{x-4}$ is $(x+4) - (x-4) = 8$. So $4(\sqrt{x+4} + \sqrt{x-4}) = 8$, and $\sqrt{x+4} + \sqrt{x-4} = 2$. Adding this to the first equation: $2\sqrt{x+4} = 6$, $x = 5$. But then $\sqrt{9} - \sqrt{1} = 2 \neq 4$. The only candidate fails, so **there is no solution**. (In fact $x \geq 4$ makes $\sqrt{x+4} + \sqrt{x-4} \geq \sqrt{8} > 2$.)

MARKING SCHEME

- ✓ The product is 8, so $\sqrt{x+4} + \sqrt{x-4} = 2$.
- ✓ The candidate $x = 5$ fails, so no solution.

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3 Long answers

a. ANSWER A · 4

$\sqrt{2x+3} = 5 - \sqrt{x+1}$. Squaring: $2x + 3 = 25 - 10\sqrt{x+1} + x + 1$, so $10\sqrt{x+1} = 23 - x$.
Squaring again: $100x + 100 = 529 - 46x + x^2$, so $x^2 - 146x + 429 = 0$, $(x - 3)(x - 143) = 0$, $x = 3$ or 143.
Check: $x = 3$: $\sqrt{4} + \sqrt{9} = 2 + 3 = 5$ ✓. $x = 143$: $\sqrt{144} + \sqrt{289} = 12 + 17 = 29$ ✗ (there $23 - x$ is negative). x **= 3.**

MARKING SCHEME

- ✓ First squaring: $10\sqrt{x+1} = 23 - x$.
- ✓ Second squaring: $x^2 - 146x + 429 = 0$.
- ✓ $x = 3$ or 143.
- ✓ Checked: $x = 3$ only.

b. ANSWER HA · 4

$L = g(T/(2\pi))^2$, from squaring $\sqrt{L/g} = T/(2\pi)$.
For 2.02 s: $L = 9.8 \times (2.02/6.2832)^2 = 9.8 \times 0.10336 \approx 1.0129$ m.
For 2 s: $L = 9.8/\pi^2 \approx 0.9929$ m.
Shorter, by about 1.0129 − 0.9929 = 0.0200 m, about 20 mm. A shorter pendulum swings faster, so the clock catches up.

MARKING SCHEME

- ✓ $L = g(T/(2\pi))^2$ by squaring.
- ✓ The present length, about 1.013 m.
- ✓ The right length, about 0.993 m.
- ✓ Shorter, by about 20 mm.

4 Think

ANSWER HA · 2

No. She only knows both answers are true because she checked them. Squaring can bring in a false root, and nothing in the squared equation shows which answer it is, so the check is the only way to know (as in $\sqrt{3x+10} = x$, where -2 fails).

MARKING SCHEME

- ✓ No: the check is what showed they are true.
- ✓ Squaring can bring in false roots that only the check finds.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	4	7	10	**21**

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