Unit 5 · Surd

ANSWER KEY · FOR TEACHERS

Full marks 31

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. surd
2. order
3. like surds
4. conjugate
5. false root
6. verified

K · 3

Each carries 0.5 mark.

2 Read the drawing

1. 297 mm
2. 148 mm

A · 2

Each carries 1 mark.

3 Multiple choice

i. **c.** $\sqrt{18}$ — K · 1

18 is not a perfect square, so $\sqrt{18} = 3\sqrt{2}$ is irrational. The others are 9, 5 and 1.2.

ii. **b.** $4\sqrt{3}$ — K · 1

$48 = 16 \times 3$, so $\sqrt{48} = 4\sqrt{3}$. ($2\sqrt{12}$ is equal but not fully simplified: 12 still has the square factor 4.)

iii. **b.** $x + 4 = 1 + 2\sqrt{x} + x$ — U · 1

$(1 + \sqrt{x})^2 = 1 + 2\sqrt{x} + x$: the middle term must not be left out.

iv. **c.** $3 + \sqrt{5}$ — U · 1

$(3 - \sqrt{5})(3 + \sqrt{5}) = 9 - 5 = 4$, a rational number.

v. **c.** $x = 9$ only — A · 1

Squaring: $x + 7 = x^2 - 10x + 25$, $x^2 - 11x + 18 = 0$, $x = 2$ or 9. At $x = 9$: $\sqrt{16} = 4 = 9 - 5$ ✓; at $x = 2$: $\sqrt{9} = 3$, but $2 - 5 = -3$ ✗.

4 Very short answers

a. MARKING SCHEME — K · 1

✓ The order is 4.

b. MARKING SCHEME — U · 1

✓ $5\sqrt{3}$.

c. MARKING SCHEME — A · 1

✓ $x = -3$.

5 Short answers

a. MARKING SCHEME — U · 2

✓ Multiplying top and bottom by $\sqrt{5} - 1$, bottom $5 - 1 = 4$.

✓ $\sqrt{5} - 1$.

b. MARKING SCHEME — K · 2

✓ $A^2 = B^2$ also holds when $A = -B$.

✓ A correct example.

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c. MARKING SCHEME A · 2

- ✓ $5x^2 = 45$, so $x = \pm 3$.
- ✓ $x = 3$ only, with -3 checked and rejected.

d. MARKING SCHEME U · 2

- ✓ The mistake: squared before separating (and lost the middle term).
- ✓ No solution, since $\sqrt{x} = -2$ is impossible.

6 Long answers

a. ANSWER A · 4

$\sqrt{x^2 - 2x - 4} = 1 + \sqrt{x^2 - 3x - 3}$. Squaring: $x^2 - 2x - 4 = 1 + 2\sqrt{x^2 - 3x - 3} + x^2 - 3x - 3$, so $x - 2 = 2\sqrt{x^2 - 3x - 3}$.
Squaring again: $x^2 - 4x + 4 = 4x^2 - 12x - 12$, $3x^2 - 8x - 16 = 0$, $(3x + 4)(x - 4) = 0$, $x = 4$ or $-4/3$.
Check: $x = 4$: $2 - 1 = 1$ ✓; $x = -4/3$: $2/3 - 5/3 = -1$ ✗. $x = \mathbf{4}$.

MARKING SCHEME

- ✓ First squaring, the root isolated: $x - 2 = 2\sqrt{x^2 - 3x - 3}$.
- ✓ Second squaring: $3x^2 - 8x - 16 = 0$.
- ✓ $x = 4$ or $-4/3$.
- ✓ Checked: $x = 4$ only.

b. ANSWER A · 4

$\sqrt{L/9.8} = 3/(2\pi)$. Squaring, $L/9.8 = 9/(4\pi^2)$, so $L = 9.8 \times 9/39.48 \approx 2.23$ m.
Check: $2\pi\sqrt{2.23/9.8} = 2\pi \times 0.477 = 3.00$ ✓.
About 2.23 m. With the clock above it and room to swing below, that is nearly as tall as a room (about 2.5–3 m), too big for most walls; that is why clocks use shorter pendulums.

MARKING SCHEME

- ✓ $\sqrt{L/9.8} = 3/(2\pi)$ (the root separated).
- ✓ Squaring: $L = 9.8 \times 9/(4\pi^2)$.
- ✓ $L \approx 2.23$ m.
- ✓ A sensible comment on its size.

7 Think

ANSWER HA · 2

$\sqrt{x - 3}$ needs $x \geq 3$. Then $x + 6 \geq 9$, so $\sqrt{x + 6} \geq 3$, and the left side is at least 3. It can never be 2.

MARKING SCHEME

- ✓ $x \geq 3$ for the first root to be real.
- ✓ Then the left side is at least 3, so it can't equal 2.

8 Activity: The shape of a sheet of paper

(not marked)

WHAT STUDENTS SHOULD FIND

Sheet	Short side (mm)	Long side (mm)	Long ÷ short
A4	210	297	1.414
A5 (half)	148	210	1.419
A6 (quarter)	105	148	1.410

Every ratio is about 1.41, which is $\sqrt{2}$: each half has the same shape as the sheet it came from. (Small differences come from the sizes being rounded to whole millimeters, and from measuring.)

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	8	7	14	2	**31**

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