Unit 4 · Quadratic Equation

HARDER WORKSHEET · ANSWER KEY

Full marks 25

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **a.** 9 — A · 1

$y = (x-3)^2 + c - 9$. It just touches the X-axis when its lowest point is on it: $c - 9 = 0$, so $c = 9$.

ii. **c.** (0, 9) — U · 1

The new parabola is $y = (x+2)^2 + 5$. At $x = 0$: $4 + 5 = 9$.

iii. **d.** -1 — A · 1

$x^2 = 2x + k$ gives $x^2 - 2x - k = 0$, that is $(x-1)^2 = 1 + k$. One solution only when $1 + k = 0$: $k = -1$, touching at (1, 1).

2 Short answers

a. ANSWER — HA · 2

$x^2 - 4x + 7 = (x-2)^2 + 3$. A square is never negative, so $(x-2)^2 + 3 \geq 3$ for every x, and it is 3 only when $x - 2 = 0$: at $x = 2$.

MARKING SCHEME

- ✓ $(x-2)^2 + 3$.
- ✓ A square is never negative, so the least value 3 is at $x = 2$.

b. MARKING SCHEME — A · 2

- ✓ $y = 2x^2 - 8x - 10$.
- ✓ Vertex $(2, -18)$.

c. MARKING SCHEME — U · 2

- ✓ The 3 must be taken out before completing the square.
- ✓ $y = 3(x+2)^2 - 7$, vertex $(-2, -7)$.

d. ANSWER — HA · 2

Any $y = a(x-3)^2 + 2$ with $a < 0$ works. For example $y = -2(x-3)^2 + 2$: $(x-3)^2 = 1$, so it cuts the X-axis at (2, 0) and (4, 0).

MARKING SCHEME

- ✓ An equation $y = a(x-3)^2 + 2$ with a negative.
- ✓ Its two x-intercepts, correct for the a chosen.

3 Long answers

a. ANSWER — A · 4

i. $A = -2x^2 + 40x = -2(x^2 - 20x) = -2\{(x-10)^2 - 100\} = -2(x-10)^2 + 200$.

ii. $a = -2 < 0$, so the vertex is the highest point: **x = 10 m, the side along the wall 20 m, area 200 m².**

iii. $-2(x-10)^2 + 200 = 150$ gives $(x-10)^2 = 25$, $x = 5$ or 15: **5 m or 15 m** (pens 5 m by 30 m and 15 m by 10 m).

MARKING SCHEME

- ✓ $A = -2(x-10)^2 + 200$.
- ✓ Largest at $x = 10$ m.
- ✓ Area 200 m².
- ✓ $x = 5$ m or 15 m.

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b. ANSWER HA · 4

i. $y = x^2 - 2x = (x-1)^2 - 1$: vertex $(1, -1)$, through $(0, 0)$, $(2, 0)$, $(-1, 3)$, $(3, 3)$, $(4, 8)$. The line $y = x + 4$ passes through $(0, 4)$, $(-1, 3)$, $(4, 8)$. **They meet at (−1, 3) and (4, 8).**

ii. $x^2 - 2x = x + k$ gives $x^2 - 3x - k = 0$, $(x - \frac{3}{2})^2 = k + \frac{9}{4}$. One solution only when $k = -\frac{9}{4}$: $x = \frac{3}{2}$, $y = \frac{3}{2} - \frac{9}{4} = -\frac{3}{4}$. **$k = -2.25$, touching at (1.5, −0.75).**

MARKING SCHEME

- ✓ The parabola drawn with its vertex $(1, -1)$ and the line $y = x + 4$.
- ✓ The meeting points $(-1, 3)$ and $(4, 8)$.
- ✓ $x^2 - 3x - k = 0$, written as a square.
- ✓ $k = -2.25$, touching at $(1.5, -0.75)$.

c. ANSWER A · 4

i. $(0, 1)$: $c = 1$. $(1, 0)$: $a + b + 1 = 0$. $(3, 4)$: $9a + 3b + 1 = 4$, so $3a + b = 1$. Subtracting: $2a = 2$, $a = 1$, $b = -2$. **$y = x^2 - 2x + 1$.**

ii. $y = (x-1)^2$: **$y = x^2$ translated 1 unit to the right** (no move up or down); vertex $(1, 0)$, touching the X-axis.

MARKING SCHEME

- ✓ $c = 1$ and the two equations in a and b.
- ✓ $a = 1$, $b = -2$.
- ✓ $y = (x-1)^2$.
- ✓ 1 unit to the right.

4 Think

ANSWER HA · 2

The first is $(x-2)^2 + 1$: lowest point $(2, 1)$. The second is $-(x-2)^2 + 1$: highest point $(2, 1)$. Everywhere else the first is above 1 and the second below 1, so they meet at one point only, $(2, 1)$, where they touch. (By algebra: $x^2 - 4x + 5 = -x^2 + 4x - 3$ gives $2(x-2)^2 = 0$.)

MARKING SCHEME

- ✓ Both in vertex form, with the same vertex $(2, 1)$, one opening up and one down.
- ✓ They touch at $(2, 1)$ only.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	12	10	**25**

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