Unit 4 · Quadratic Equation

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. quadratic
2. parabola
3. vertex
4. axis of symmetry
5. completing the square
6. translation

K · 3

Each carries 0.5 mark.

2 Read the graph

1. 1 s
2. highest: 20 m at 2 s
3. 3 s

U · 3

Each carries 1 mark.

3 Multiple choice

i. **c.** $x^2 = 4x$ — K · 1

$x^2 = 4x$ is $x^2 - 4x = 0$: the highest power of x is 2.

ii. **a.** $(-5, -2)$ — K · 1

The bracket is 0 when $x = -5$, and then $y = -2$.

iii. **b.** opens downward, with highest point (1, 3) — U · 1

$a = -2 < 0$: it opens downward, so the vertex (1, 3) is its highest point.

iv. **d.** 4 units to the right — U · 1

$x - 4$ in the bracket moves the parabola 4 units to the right: its vertex goes from (0, 0) to (4, 0).

v. **c.** $x = 1$ and $x = 5$ — A · 1

$x^2 - 6x + 5 = (x - 1)(x - 5)$, which is 0 at $x = 1$ and $x = 5$.

4 Very short answers

a. MARKING SCHEME — K · 1

✓ $x = 2$.

b. MARKING SCHEME — U · 1

✓ Never, because its lowest point is above the X-axis.

c. MARKING SCHEME — A · 1

✓ $y = (x + 2)^2 - 6$.

5 Short answers

a. MARKING SCHEME — U · 2

✓ $y = (x + 4)^2 - 6$.

✓ Vertex $(-4, -6)$.

b. MARKING SCHEME — U · 2

✓ 3 units to the left, not the right.

✓ Vertex $(-3, 1)$.

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c. MARKING SCHEME K · 2

- ✓ Points on the X-axis have $y = 0$.
- ✓ There $ax^2 + bx + c = 0$, so their x are the solutions.

d. MARKING SCHEME A · 2

- ✓ $y = a(x - 1)^2 - 4$ with the vertex put in.
- ✓ $a = 1$: $y = (x - 1)^2 - 4$.

6 Long answers

a. ANSWER A · 4

$y = x^2 - 2x + 1 - 1 - 8 = (x - 1)^2 - 9$. Opens upward; vertex $(1, -9)$; axis $x = 1$.
Y-axis: $x = 0$ gives $y = -8$, the point $(0, -8)$, with its mirror point $(2, -8)$.
X-axis: $(x - 1)^2 = 9$, $x - 1 = \pm 3$, so $x = 4$ or -2: $(4, 0)$ and $(-2, 0)$.
The sketch: a $\cup$ through these points, round at $(1, -9)$, symmetric about $x = 1$.

MARKING SCHEME

- ✓ $y = (x - 1)^2 - 9$.
- ✓ Vertex $(1, -9)$ and axis $x = 1$.
- ✓ Intercepts $(0, -8)$, $(-2, 0)$ and $(4, 0)$.
- ✓ A smooth $\cup$ through them, symmetric about $x = 1$.

b. ANSWER A · 4

Table for $y = x^2$: $x = -3, -2, -1, 0, 1, 2, 3$ give $9, 4, 1, 0, 1, 4, 9$. The line $y = x + 6$ passes through $(0, 6)$, $(-2, 4)$ and $(3, 9)$.
They meet at $(-2, 4)$ and $(3, 9)$. **The solutions are $x = -2$, $y = 4$ and $x = 3$, $y = 9$.**
Check: $x^2 = x + 6$ gives $x^2 - x - 6 = 0$, $(x - 3)(x + 2) = 0$, so $x = 3$ or -2.

MARKING SCHEME

- ✓ The table and the parabola $y = x^2$ drawn.
- ✓ The line $y = x + 6$ drawn.
- ✓ The meeting points $(-2, 4)$ and $(3, 9)$ read off.
- ✓ The algebra check.

7 Think

ANSWER HA · 2

After 3 seconds. The graph of h is a parabola, symmetric about the line through its highest point, so the two times at the same height are the same distance either side of it: halfway between 1 and 5 is 3.
(Check: $h = -5(t - 3)^2 + 45$.)

MARKING SCHEME

- ✓ 3 seconds.
- ✓ Because the parabola is symmetric: the axis is halfway between the two times at the same height.

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8 Activity: A parabola from a table

(not marked)

WHAT STUDENTS SHOULD FIND

x	$y = x^2 - 4x + 3$
-1	8
0	3
1	0
2	-1
3	0
4	3
5	8

The lowest point is $(2, -1)$. Points the same distance either side of $x = 2$ are at the same height (0 and 4, -1 and 5, 1 and 3): the graph is symmetric about $x = 2$. It cuts the X-axis at $x = 1$ and $x = 3$, the solutions of $x^2 - 4x + 3 = 0$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	8	10	12	2	**32**

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