Unit 3 · Linear Programming

HARDER WORKSHEET · ANSWER KEY

Full marks 26

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **b.** $x + y \leq 1, x + y \geq 3$ — A · 1

$x + y$ can't be at most 1 and at least 3 at once: the two half-planes don't overlap. The others all have points, such as (1, 1), (1, −1) and (2, −2).

ii. **c.** 7 — HA · 1

Z is 0, 8, 10 and 10 at the vertices: B and C tie, so every point of the edge BC, on $x + y = 10$, gives 10. Its whole-number points are (6, 4), (5, 5), (4, 6), (3, 7), (2, 8), (1, 9) and (0, 10): seven. The points of the edge are infinitely many, but only seven are whole-number plans.

iii. **b.** $x \leq 6$ — A · 1

With $x + y \leq 4$ and $y \geq 0$, x is already at most 4, so $x \leq 6$ cuts nothing off.

iv. **c.** $2x - 3y < 6$ — U · 1

$\frac{x}{3} + \frac{y}{-2} = 1$ gives $2x - 3y = 6$. O is above the line and gives $0 < 6$; dashed means $<$.

2 Short answers

a. MARKING SCHEME — A · 2

- ✓ The three boundary equations.
- ✓ The three signs from a point inside.

b. MARKING SCHEME — U · 2

- ✓ (8, 0) and (0, 6) are not in the region, each with the constraint it breaks.
- ✓ The vertices $O(0, 0)$, (6, 0), (4, 2), (0, 4).

c. MARKING SCHEME — A · 2

- ✓ $800 > 750$, so the report is not possible.
- ✓ At most 18 copies.

d. ANSWER — HA · 2

$x \geq 1, x \leq 3, y \geq 1, y \leq 3$ (or $1 \leq x \leq 3, 1 \leq y \leq 3$). Z at the corners: 1, 5, 3, −1. **The maximum is 5, at (3, 1).**

MARKING SCHEME

- ✓ The four inequalities.
- ✓ Maximum 5 at (3, 1).

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3 Long answers

a. ANSWER A · 4

Let x be the chairs and y the tables. Carpentry $2x + 3y \leq 24$, polishing $x + y \leq 10$, room $y \leq 6$, and $x, y \geq 0$. Maximize $Z = 1500x + 2500y$.
Corners: $O(0, 0)$; $A(10, 0)$; B, where $2x + 3y = 24$ meets $x + y = 10$: $y = 4$, $x = 6$; C, where $2x + 3y = 24$ meets $y = 6$: $x = 3$; $D(0, 6)$.
Z: 0, 15,000, 19,000, 19,500, 15,000.
3 chairs and 6 tables, for Rs 19,500.

MARKING SCHEME

- ✓ The constraints and the objective function.
- ✓ The feasible region drawn, with its five vertices.
- ✓ $B(6, 4)$ and $C(3, 6)$ found by solving.
- ✓ 3 chairs and 6 tables, Rs 19,500.

b. ANSWER HA · 4

i. $x + y = 5$ and $2x + y = 8$ meet where $x = 3$, $y = 2$. The vertices are $O(0, 0)$, $(4, 0)$, $(3, 2)$ and $(0, 5)$.
ii. Z is 0, $4p$, $3p + 2$ and 5. Need $3p + 2 \geq 4p$, so $p \leq 2$, and $3p + 2 \geq 5$, so $p \geq 1$ (and then it is also more than 0). So $1 \leq p \leq 2$. At $p = 1$ or $p = 2$ it ties with a neighboring vertex.

MARKING SCHEME

- ✓ The four vertices, with (3, 2) found by solving.
- ✓ Z at each vertex in terms of p.
- ✓ $3p + 2 \geq 4p$ gives $p \leq 2$.
- ✓ $3p + 2 \geq 5$ gives $p \geq 1$: so $1 \leq p \leq 2$.

c. ANSWER A · 4

$x + y = 5$ through (5, 0) and (0, 5); $x + 3y = 9$ through (9, 0) and (0, 3). O gives false for both, so the region is on the side away from O: an open region.
The lines meet where $2y = 4$: (3, 2). Corners (0, 5), (3, 2), (9, 0); Z: 10, 13, 27.
The minimum is 10, at (0, 5). There is **no maximum**: the region runs on without end, and Z grows without limit (at (100, 0) it is 300).

MARKING SCHEME

- ✓ The region drawn on the side away from O.
- ✓ The corners, with (3, 2) found by solving.
- ✓ Minimum 10 at (0, 5).
- ✓ No maximum, because the region is open.

4 Think

ANSWER HA · 2

Not safely: a rounded point may break a constraint (it may need more flour or oven time than there is) and need not be the best whole-number plan. It should check the whole-number points of the region near the corner, keep only those that satisfy every constraint, and work out the profit at each to pick the best.

MARKING SCHEME

- ✓ Rounding can leave the feasible region, or miss the best whole-number plan.
- ✓ Check the nearby whole-number points that keep every constraint, and compare their values.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	14	9	**26**

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