Unit 3 · Linear Programming

ANSWER KEY · FOR TEACHERS

Full marks 29

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. inequality
2. boundary line
3. test point
4. feasible region
5. constraints
6. objective function

K · 3

Each carries 0.5 mark.

2 Multiple choice

i. **b.** $x \leq 40$ — K · 1

"At most 40" means 40 or fewer: $x \leq 40$.

ii. **d.** dashed, and the region does not contain O — U · 1

$>$ leaves out the line, so it is dashed. O gives $0 > 12$, false, so O is not in the region.

iii. **b.** (3, 1) — A · 1

(3, 1): $4 \leq 5$, $2 \geq 1$, $1 \geq 0$. (0, 0) fails $x - y \geq 1$; (2, 2) gives $0 \geq 1$, false; (4, 2) gives $6 \leq 5$, false.

iv. **c.** 25 — A · 1

Z is 0 at O, 12 at A, $8 + 15 = 23$ at B and 25 at C. The maximum is 25, at C.

v. **b.** $x \leq -2$ — U · 1

Points to the left of $x = -2$ have x smaller than -2, and "the line included" makes it $\leq$.

3 Very short answers

a. MARKING SCHEME — K · 1

✓ $x \geq 18$.

b. MARKING SCHEME — U · 1

✓ $8 < 8$ is false, so (2, 3) is not in the region.

c. MARKING SCHEME — A · 1

✓ (4, 2).

4 Short answers

a. MARKING SCHEME — U · 2

✓ A false test means the region is on the other side.

✓ The side not containing O.

b. MARKING SCHEME — K · 2

✓ The region allowed by all the inequalities together.

✓ The vertices belong to the region, so they satisfy all of them.

c. MARKING SCHEME — U · 2

✓ Each value of Z is a line, moving parallel to itself as the value grows.

✓ It last touches the region at a vertex (or along an edge ending in vertices).

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d. MARKING SCHEME A · 2

- ✓ The boundary's equation $3x + 2y = 6$ from the two intercepts.
- ✓ The sign from a test point: $3x + 2y \leq 6$.

5 Long answers

a. ANSWER A · 4

$x + 2y = 10$ through (10, 0) and (0, 5); $3x + y = 15$ through (5, 0) and (0, 15). O satisfies both, so both regions contain O.
They meet where, from $x = 10 - 2y$, $30 - 6y + y = 15$: $y = 3$, $x = 4$.
Vertices $O(0, 0)$, $A(5, 0)$, $B(4, 3)$, $C(0, 5)$. Z: 0, 20, 16 + 15 = 31, 25.
Maximum 31 at (4, 3); minimum 0 at (0, 0).

MARKING SCHEME

- ✓ Both lines drawn correctly (solid), with $x \geq 0, y \geq 0$.
- ✓ Feasible region $OABC$ shaded.
- ✓ Vertex $B(4, 3)$ found by solving the two equations.
- ✓ Maximum 31 at (4, 3), minimum 0 at (0, 0).

b. ANSWER A · 4

Let x ropani be potatoes and y cauliflower. Land: $x + y \leq 8$. Seed: $2000x + 1000y \leq 12000$, that is $2x + y \leq 12$. $x, y \geq 0$. Maximize $Z = 5000x + 3000y$.
$x + y = 8$ through (8, 0), (0, 8); $2x + y = 12$ through (6, 0), (0, 12). They meet at $x = 4$, $y = 4$.
Vertices $O(0, 0)$, $A(6, 0)$, $B(4, 4)$, $C(0, 8)$. Z: 0, 30,000, 32,000, 24,000.
4 ropani of potatoes and 4 of cauliflower, for Rs 32,000, using all the land and all the seed money.

MARKING SCHEME

- ✓ The constraints $x + y \leq 8$, $2x + y \leq 12$, $x, y \geq 0$ and $Z = 5000x + 3000y$.
- ✓ The feasible region drawn and shaded.
- ✓ The vertices, with $B(4, 4)$ found by solving.
- ✓ 4 ropani of each, Rs 32,000.

6 Think

ANSWER HA · 2

A minimum, yes: 8, at (4, 0) (at (0, 4) it is 12). A maximum, no: the region is open and runs on without end, so x and y can be as large as you like, and so can Z (at (100, 0) it is already 200). The corner rule finds a maximum only when one exists.

MARKING SCHEME

- ✓ Minimum 8 at (4, 0).
- ✓ No maximum, because the region is open and Z grows without limit.

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7 Activity: One point decides the side

(not marked)

WHAT STUDENTS SHOULD FIND

Point	$2x + y$	True or false?	Side
(0, 0)	0	true	O's side
(1, 2)	4	true	O's side
(3, 0)	6	true	on the line
(2, 3)	7	false	the other side
(−1, 5)	3	true	O's side
(4, −1)	7	false	the other side

Every true point is on O's side or on the line (solid, since $\leq$); every false one is on the other side. So one test point is enough to decide which side to shade.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	7	7	13	2	**29**

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