Unit 2 · Polynomial

HARDER WORKSHEET

Full marks 26

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 26

1 Multiple choice [4]

Choose the correct option for each of the following questions.

i. What is the remainder when $x^{101} + 2$ is divided by $x + 1$? [1]

a. 1 **b.** 3

c. 2 **d.** 0

ii. $x^2 - 1$ is a factor of $x^3 + ax^2 + bx + 2$. What are a and b? [1]

a. $a = -2, b = -1$ **b.** $a = 2, b = 1$

c. $a = -1, b = -2$ **d.** $a = 1, b = -3$

iii. $f(x)$ leaves a remainder of 5 when divided by $x - 2$. What remainder does $x \cdot f(x)$ leave when divided by $x - 2$? [1]

a. 5 **b.** 10

c. 7 **d.** 0

iv. A cubic with leading coefficient 1 has roots 1, 2 and -3. What is its constant term? [1]

a. 6 **b.** -6

c. 0 **d.** 1

2 Short answers [8]

Write short answers to the following questions.

a. Work backwards. Dividing a polynomial by $x - 2$ by synthetic division gave the bottom row 3, 1, -4, 5. Find the polynomial. [2]

b. Show that $x - 1$ is a factor of $x^n - 1$ for every whole number $n \geq 1$, and that $x + 1$ is a factor of $x^n + 1$ whenever n is odd. [2]

c. Find the mistake. Ram lists the possible rational roots of $3x^3 - x^2 + 2x - 4$ as $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}$. What has he done wrong? Write the correct list. **[2]**

d. Make your own. Write a cubic with whole-number coefficients whose roots are 2, -1 and $\frac{1}{3}$. Check one of its roots with the remainder theorem. **[2]**

3 Long answers [12]

Solve the following problems. Show your working.

a. A polynomial $f(x)$ leaves a remainder of 3 when divided by $x - 1$ and a remainder of 5 when divided by $x - 2$. **[4]**

i. Find the remainder when $f(x)$ is divided by $(x - 1)(x - 2)$.

ii. Give one such polynomial of degree 2.

Answer:

b. Is it possible? Without working out $f\left(\frac{1}{3}\right)$, explain why $\frac{1}{3}$ can not be a root of $f(x) = 2x^3 + 3x^2 - 1$. **[4]**

Then factorize $f(x)$ completely.

Answer: _____

c. A ward office builds a water tank in the shape of a cuboid. Its length is 2 m more than its height, and its width is 1 m less than its height. It must hold 30 m^3 (30,000 liters). **[4]**

i. Taking the height as h m, show that $h^3 + h^2 - 2h - 30 = 0$.

ii. Find the tank's three measurements, and show that there is no other answer.

Answer: _____

4 Think **[2]**

Think, then answer.

Without dividing, how can you tell that $x^4 + 3x^2 + 7$ has no factor $x - a$ for any number a?