Unit 2 · Polynomial

HARDER WORKSHEET · ANSWER KEY

Full marks 26

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **a.** 1 — A · 1

$(-1)^{101} = -1$, since 101 is odd, so the remainder is $-1 + 2 = 1$. 3 comes from putting in $x = 1$.

ii. **a.** $a = -2, b = -1$ — A · 1

$x^2 - 1 = (x - 1)(x + 1)$, so both $f(1)$ and $f(-1)$ are 0: $a + b = -3$ and $a - b = -1$, giving $a = -2$, $b = -1$.

iii. **b.** 10 — HA · 1

The remainder is the value at $x = 2$: $2 \cdot f(2) = 2 \times 5 = 10$.

iv. **a.** 6 — A · 1

It is $(x - 1)(x - 2)(x + 3)$, and its constant term is $(-1)(-2)(3) = 6$.

2 Short answers

a. MARKING SCHEME — A · 2

- ✓ Quotient $3x^2 + x - 4$ and remainder 5 read from the row.
- ✓ $f(x) = 3x^3 - 5x^2 - 6x + 13$.

b. MARKING SCHEME — U · 2

- ✓ $f(1) = 1^n - 1 = 0$ for every n.
- ✓ $g(-1) = (-1)^n + 1 = 0$ because n is odd.

c. MARKING SCHEME — U · 2

- ✓ p must divide the constant term (4), q the leading coefficient (3).
- ✓ The correct list $\pm1, \pm2, \pm4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$.

d. ANSWER — HA · 2

$(x - 2)(x + 1)(3x - 1) = (x^2 - x - 2)(3x - 1) = 3x^3 - 4x^2 - 5x + 2$. (The factor for $\frac{1}{3}$ is written $3x - 1$ to keep the coefficients whole.)

Check: $f(2) = 24 - 16 - 10 + 2 = 0$. ✓ (Any whole-number multiple of it is right too.)

MARKING SCHEME

- ✓ $3x^3 - 4x^2 - 5x + 2$, or a multiple of it.
- ✓ One root checked to give 0.

3 Long answers

a. ANSWER — HA · 4

i. $f(x) = (x - 1)(x - 2) \cdot q(x) + ax + b$. Put $x = 1$: $f(1) = a + b = 3$. Put $x = 2$: $f(2) = 2a + b = 5$. Subtracting, $a = 2$, and $b = 1$. The remainder is $2x + 1$.

ii. With $q(x) = 1$: $f(x) = (x - 1)(x - 2) + 2x + 1 = x^2 - x + 3$. Check: $f(1) = 3$ ✓, $f(2) = 5$ ✓.

MARKING SCHEME

- ✓ The remainder written as $ax + b$, since it has a lower degree than the divisor.
- ✓ $a + b = 3$ and $2a + b = 5$.
- ✓ Remainder $2x + 1$.
- ✓ A correct polynomial, such as $x^2 - x + 3$.

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b. **ANSWER** HA · 4

A rational root $\frac{p}{q}$ in lowest terms has q dividing the leading coefficient 2, so its denominator is 1 or 2: a third is impossible. The candidates are ± 1, $\pm \frac{1}{2}$.
$f(1) = 4$, but $f(-1) = -2 + 3 - 1 = 0$, so $x + 1$ is a factor. Synthetic division by -1 of 2, 3, 0, -1 gives 2, 1, -1, 0: the quotient is $2x^2 + x - 1 = (2x - 1)(x + 1)$.
So $f(x) = (x + 1)^2(2x - 1)$: the roots are -1 (twice) and $\frac{1}{2}$.

MARKING SCHEME

- ✓ q must divide 2, so a root with denominator 3 is impossible.
- ✓ $f(-1) = 0$, so $x + 1$ is a factor.
- ✓ Quotient $2x^2 + x - 1$.
- ✓ $f(x) = (x + 1)^2(2x - 1)$.

c. **ANSWER** A · 4

i. $h(h + 2)(h - 1) = 30$, and $(h + 2)(h - 1) = h^2 + h - 2$, so $h^3 + h^2 - 2h = 30$, which is $h^3 + h^2 - 2h - 30 = 0$.

ii. $f(3) = 27 + 9 - 6 - 30 = 0$, so $h - 3$ is a factor. Synthetic division by 3 of 1, 1, -2, -30 gives 1, 4, 10, 0: $f(h) = (h - 3)(h^2 + 4h + 10)$.

$h^2 + 4h + 10 = (h + 2)^2 + 6$ is never 0 (its discriminant $16 - 40$ is negative), so $h = 3$ is the only answer.
The tank is 5 m long, 2 m wide and 3 m high: $5 \times 2 \times 3 = 30$ m^3. ✓

MARKING SCHEME

- ✓ The equation formed from $h(h + 2)(h - 1) = 30$.
- ✓ $f(3) = 0$, so $h - 3$ is a factor.
- ✓ $h^2 + 4h + 10$ has no real root, so $h = 3$ only.
- ✓ 5 m by 2 m by 3 m.

4 Think

ANSWER HA · 2

For any number a, $a^4 \geq 0$ and $3a^2 \geq 0$, so $f(a) = a^4 + 3a^2 + 7 \geq 7$. It is never 0, so by the factor theorem no $x - a$ is a factor. (Its graph stays above the X-axis.)

MARKING SCHEME

- ✓ $f(a) \geq 7$ for every a, so $f(a)$ is never 0.
- ✓ So, by the factor theorem, no $x - a$ is a factor.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	4	9	13	**26**

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