Unit 1 · Function

HARDER WORKSHEET

Full marks 27

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 27

1 Multiple choice [4]

Choose the correct option for each of the following questions.

i. $f(x) = ax + b$ with $a > 0$, and $f \circ f(x) = 4x + 9$. What is $f(x)$? [1]

a. $4x + 9$ **b.** $2x + 3$

c. $2x + 9$ **d.** $2x + \frac{9}{2}$

ii. $f(x) = 3x + 1$ and $g \circ f(x) = 6x - 1$. What is $g(x)$? [1]

a. $2x - 1$ **b.** $3x - 2$

c. $2x - 3$ **d.** $6x - 4$

iii. If $f(x) = 2x + 3$, what is $f^{-1} \circ f^{-1}(x)$? [1]

a. $\frac{x - 6}{4}$ **b.** $\frac{x - 9}{4}$

c. $\frac{x - 3}{4}$ **d.** $\frac{1}{4x + 9}$

iv. $f(x) = \frac{x + 1}{x - 1}$ for $x \neq 1$. What is $f^{-1}(x)$? [1]

a. $\frac{x + 1}{x - 1}$ **b.** $\frac{x - 1}{x + 1}$

c. $\frac{1 - x}{1 + x}$ **d.** it has no inverse

2 Short answers [10]

Write short answers to the following questions. Show your working.

a. Work backwards. $f(x) = 3x - 4$ and $g(x) = x^2$. If $g \circ f(k) = 25$, find every possible value of k. [2]

b. Is it possible? Sita writes: "$f(x) = x^2 + 1$, on all real numbers, has the inverse $f^{-1}(x) = \sqrt{x - 1}$." Is she right? If not, how can it be put right? [2]

c. $f(x) = \dfrac{3x-2}{x+4}$, $x \neq -4$. Find $f^{-1}(x)$, the one value of x where f^{-1} is not defined, and check your answer with $x = 0$. **[3]**

d. $f(x) = x + 2$ and $g(x) = 3x$. Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$, and explain in words why the order turns round. **[3]**

3 Long answers **[11]**

Solve the following problems. Show your working.

a. Suppose a taxi in a town charges Rs. 50 to start and Rs. 40 for each kilometer, so the fare for d km is $f(d) = 50 + 40d$. At night the fare is one and a half times as much: $g(r) = 1.5r$. **[4]**

i. Find $g \circ f(d)$, the night fare for d km, and the night fare for 5 km.

ii. Find $f^{-1}(r)$. How far did a passenger go who paid Rs. 330 by day?

iii. A passenger paid Rs. 600 at night. How far did she go?

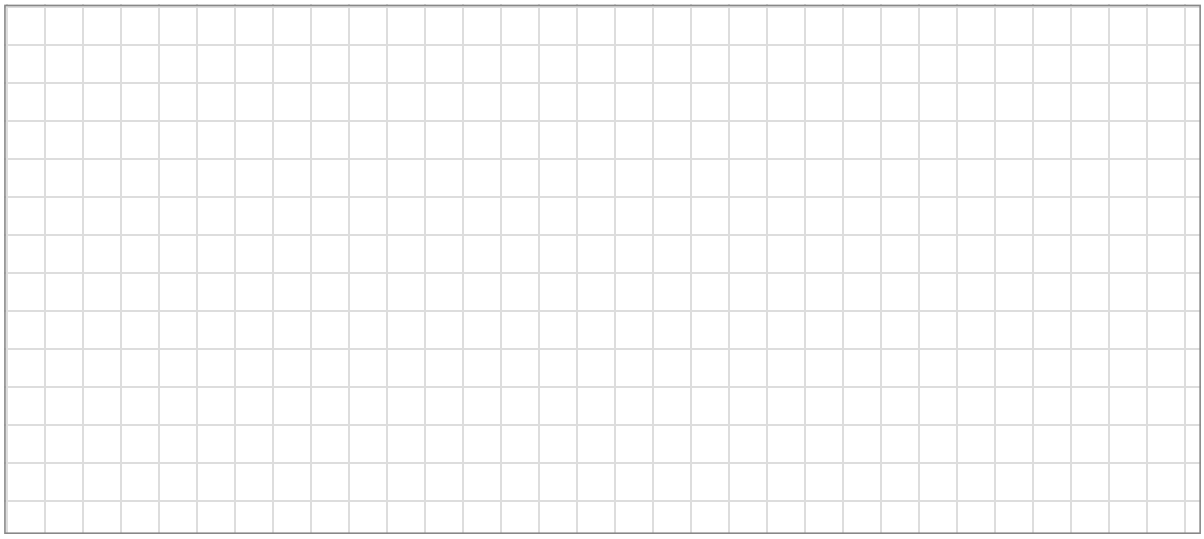
Answer:

b. Make your own. $f(x) = 2x + 1$ and $g(x) = ax + b$. **[4]**

i. Find the condition on a and b for $g \circ f = f \circ g$.

ii. Give two different functions g that fit.

iii. Does f^{-1} fit? Show it.



Answer: _____

c. Show that $f(x) = \frac{x+3}{x-1}$ $(x \neq 1)$ is its own inverse. Then find the points where its graph meets the line $y = x$. **[3]**

4 Think **[2]**

Think, then answer.

In a Caesar code each letter moves 3 places along the alphabet, going round from Z back to A (A $\rightarrow$ D, X $\rightarrow$ A). A message is coded with a move of 3 and then with a move of 5. Which single move does both, and which move undoes it? How many moves of 3 bring every letter back to itself?