Unit 1 · Function

HARDER WORKSHEET · ANSWER KEY

Full marks 27

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **b.** $2x + 3$ **A · 1**

$f(f(x)) = a(ax + b) + b = a^2x + ab + b$. So $a^2 = 4$, $a = 2$, and $2b + b = 9$, $b = 3$.

ii. **c.** $2x - 3$ **A · 1**

$g(3x + 1) = 6x - 1 = 2(3x + 1) - 3$, so g doubles and takes away 3: $g(x) = 2x - 3$. Check: $g(f(x)) = 6x + 2 - 3 = 6x - 1$.

iii. **b.** $\dfrac{x - 9}{4}$ **A · 1**

$f^{-1}(x) = \dfrac{x - 3}{2}$, and again: $\dfrac{\frac{x-3}{2} - 3}{2} = \dfrac{x - 9}{4}$. It is also the inverse of $f \circ f(x) = 4x + 9$.

iv. **a.** $\dfrac{x + 1}{x - 1}$ **HA · 1**

$f(f(x)) = \dfrac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = \dfrac{2x}{2} = x$, so f undoes itself: it is its own inverse.

2 Short answers

a. **MARKING SCHEME** **A · 2**

- ✓ $3k - 4 = \pm 5$.
- ✓ $k = 3$ or $k = -\dfrac{1}{3}$.

b. **MARKING SCHEME** **U · 2**

- ✓ Not right: two numbers (2 and -2) have the same image, so there is no inverse.
- ✓ Restricted to $x \geq 0$, the inverse is $\sqrt{x - 1}$.

c. **MARKING SCHEME** **A · 3**

- ✓ $x = \dfrac{3y - 2}{y + 4}$ rearranged for y.
- ✓ $f^{-1}(x) = \dfrac{4x + 2}{3 - x}$, $x \neq 3$.
- ✓ The check with 0 (or another number) done.

d. **ANSWER** **HA · 3**

$g \circ f(x) = 3(x + 2) = 3x + 6$, so $(g \circ f)^{-1}(x) = \dfrac{x - 6}{3} = \dfrac{x}{3} - 2$. Also $g^{-1}(x) = \dfrac{x}{3}$ and $f^{-1}(x) = x - 2$, so $f^{-1} \circ g^{-1}(x) = \dfrac{x}{3} - 2$. They are equal. In $g \circ f$, f acts first and g last; to undo it, the last step (g) is undone first, as shoes come off before socks.

MARKING SCHEME

- ✓ $(g \circ f)^{-1}(x) = \dfrac{x}{3} - 2$.
- ✓ $f^{-1} \circ g^{-1}(x) = \dfrac{x}{3} - 2$, the same.
- ✓ The reason: the last step is undone first.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

3 Long answers

a. **ANSWER** **A · 4**

i. $g \circ f(d) = 1.5(50 + 40d) = 75 + 60d$; for 5 km, $75 + 300 =$ Rs. 375.

ii. $r = 50 + 40d$ gives $f^{-1}(r) = \dfrac{r - 50}{40}$; $f^{-1}(330) = \dfrac{280}{40} = 7$ km.

iii. $(g \circ f)^{-1}(600) = f^{-1}(g^{-1}(600)) = f^{-1}(400) = \dfrac{350}{40} = 8.75$ km (or solve $75 + 60d = 600$).

MARKING SCHEME

- ✓ $g \circ f(d) = 75 + 60d$.
- ✓ Night fare for 5 km Rs. 375.
- ✓ $f^{-1}(r) = \dfrac{r - 50}{40}$, 7 km.
- ✓ 8.75 km at night.

b. **ANSWER** **HA · 4**

i. $g \circ f(x) = a(2x + 1) + b = 2ax + a + b$; $f \circ g(x) = 2(ax + b) + 1 = 2ax + 2b + 1$. Equal when $a + b = 2b + 1$, that is $a = b + 1$.

ii. For example $b = 0$, $a = 1$: $g(x) = x$; $b = 2$, $a = 3$: $g(x) = 3x + 2$. (Check: $g \circ f(x) = 6x + 5 = f \circ g(x)$.)

iii. $f^{-1}(x) = \dfrac{x - 1}{2}$, so $a = \dfrac{1}{2}$, $b = -\dfrac{1}{2}$, and $a = b + 1$ ✓: a function and its inverse always give the identity in either order.

MARKING SCHEME

- ✓ Both composites worked out.
- ✓ The condition $a = b + 1$.
- ✓ Two correct functions g.
- ✓ $f^{-1}(x) = \dfrac{x - 1}{2}$ fits the condition.

c. **ANSWER** **HA · 3**

$f(f(x)) = \dfrac{\dfrac{x + 3}{x - 1} + 3}{\dfrac{x + 3}{x - 1} - 1} = \dfrac{\dfrac{4x}{x - 1}}{\dfrac{4}{x - 1}} = x$, so f undoes itself: $f^{-1} = f$. On $y = x$: $\dfrac{x + 3}{x - 1} = x$, $x^2 - 2x - 3 = 0$, $(x - 3)(x + 1) = 0$: the points $(3, 3)$ and $(-1, -1)$.

MARKING SCHEME

- ✓ $f(f(x)) = x$ shown.
- ✓ $x^2 - 2x - 3 = 0$ formed.
- ✓ $(3, 3)$ and $(-1, -1)$.

4 Think

ANSWER **HA · 2**

The two moves together are one move of 8; it is undone by a move of 8 back (or 18 forward, since $8 + 18 = 26$). Moves of 3 bring a letter back only when the total is a whole number of rounds of 26: $3 \times 26 = 78$ is the first multiple of 3 that is also a multiple of 26, so **26 moves** of 3.

MARKING SCHEME

- ✓ A move of 8, undone by a move of 8 back (18 forward).
- ✓ 26 moves of 3.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	2	12	13	**27**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking