Unit 1 · Function

ANSWER KEY · FOR TEACHERS

Full marks 31

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. composite
2. domain
3. identity
4. one-to-one onto
5. inverse
6. interchanges

Each carries 0.5 mark. K · 3

2 Multiple choice

i. **a.** $\frac{x+2}{3}$ K · 1

f multiplies by 3, then takes away 2. Undo the last step first: add 2, then divide by 3. $\frac{1}{3x-2}$ is the reciprocal, not the inverse.

ii. **b.** $\{(1, 5), (2, 6), (3, 4)\}$ U · 1

Only the second is one-to-one onto: each element of B is the image of exactly one element of A. The others send two elements to the same image and miss an element of B.

iii. **b.** 4 A · 1

In $f \circ g$, g acts first: $g(3) = 2$, then $f(2) = 4$. The answer 8 is $g \circ f(3)$, the other order.

iv. **c.** 2 U · 1

f takes 2 to 7, so f^{-1} takes 7 back to 2.

v. **c.** $4x + 3$ A · 1

$f(f(x)) = 2(2x + 1) + 1 = 4x + 3$. $4x^2 + 4x + 1$ is $f(x) \times f(x)$, a product, not a composite.

3 Very short answers

a. MARKING SCHEME U · 1

✓ $5x + 1$ found.

b. MARKING SCHEME K · 1

✓ $\{(a, 1), (b, 2), (c, 3)\}$ written.

c. MARKING SCHEME A · 1

✓ $f(x) = 4x + 1$.

4 Short answers

a. MARKING SCHEME U · 2

✓ The order is wrong: in $g \circ f$ the function f comes first.

✓ $g \circ f(2) = 10$ found.

b. MARKING SCHEME U · 2

✓ $g \circ f = \{(1, 5), (2, 3), (3, 2), (5, 1)\}$.

✓ $f \circ g = \{(1, 3), (2, 5), (3, 1), (5, 2)\}$.

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c. MARKING SCHEME A · 2

✓ $f^{-1}(x) = \dfrac{5x-3}{2}$.

✓ $f^{-1}(3) = 6$.

d. MARKING SCHEME U · 2

✓ $f(g(x)) = x$ shown.

✓ $g(f(x)) = x$ shown, so they are inverses.

e. MARKING SCHEME U · 2

✓ Not one-to-one: two numbers (such as 2 and -2) have the same square.

✓ On $x \geq 0$ it has an inverse ($\sqrt{x}$).

5 Long answers

a. ANSWER A · 4

i. $g \circ f(x) = g(2x-1) = (2x-1)^2 + 1 = 4x^2 - 4x + 2$.

ii. $f \circ g(x) = f(x^2+1) = 2(x^2+1) - 1 = 2x^2 + 1$. They are not equal (try $x = 0$: 2 and 1).

iii. $4x^2 - 4x + 2 = 10$, so $x^2 - x - 2 = 0$, $(x-2)(x+1) = 0$: $x = 2$ or $x = -1$. Check: $f(2) = 3$, $g(3) = 10$ ✓; $f(-1) = -3$, $g(-3) = 10$ ✓.

MARKING SCHEME

✓ $g \circ f(x) = 4x^2 - 4x + 2$.

✓ $f \circ g(x) = 2x^2 + 1$, not equal to $g \circ f(x)$.

✓ The equation $x^2 - x - 2 = 0$ formed.

✓ $x = 2$ or $x = -1$.

b. ANSWER A · 4

i. Let $y = \dfrac{9x}{5} + 32$; interchange: $x = \dfrac{9y}{5} + 32$, so $y = \dfrac{5(x-32)}{9}$. $f^{-1}(x) = \dfrac{5(x-32)}{9}$ turns °F back into °C.

ii. $f^{-1}(104) = \dfrac{5 \times 72}{9} = 40$: 40 °C, a high fever.

iii. $\dfrac{9x}{5} + 32 = x$ gives $\dfrac{4x}{5} = -32$, so $x = -40$: -40 °C is -40 °F.

iv. At $(-40, -40)$, on the line $y = x$: the point where $f(x) = x$ is its own mirror image.

MARKING SCHEME

✓ $f^{-1}(x) = \dfrac{5(x-32)}{9}$, changing °F to °C.

✓ 104 °F = 40 °C.

✓ -40 found from $f(x) = x$.

✓ The graphs meet at $(-40, -40)$, on $y = x$.

6 Think

ANSWER HA · 2

For example $f(x) = 6 - x$: $f(f(x)) = 6 - (6 - x) = x$, so applying it twice gives back x, and f undoes itself. (Also $f(x) = -x$, or $f(x) = \dfrac{1}{x}$ for $x \neq 0$.) Its graph is its own mirror image in $y = x$.

MARKING SCHEME

✓ A correct function given (such as $6 - x$, $-x$ or $\dfrac{1}{x}$).

✓ $f(f(x)) = x$ shown for it.

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7 Activity: Rules and their undo

(not marked)

WHAT STUDENTS SHOULD FIND

Rule	$f(3)$	**Undo**	$f^{-1}(x)$	**Back**
$2x + 5$	11	-5, then $\div 2$	$\frac{x-5}{2}$	3
$\frac{x-4}{3}$	$-\frac{1}{3}$	$\times 3$, then $+4$	$3x + 4$	3
$10 - x$	7	take it from 10	$10 - x$	3
$x^3 + 1$	28	-1, then the cube root	$\sqrt[3]{x-1}$	3
Each inverse undoes the steps in the reverse order, and always brings 3 back. $10 - x$ is its own inverse.				

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	11	13	2	**31**

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