Unit 15 · Trigonometry

HARDER WORKSHEET · ANSWER KEY

Full marks 22

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 A tower between two points

ANSWER HA · 3

Let the height be h and the distance from P be x, so from Q it is $80 - x$.

$h = x \tan 60^\circ = \sqrt{3}\,x$ and $h = (80 - x)\tan 30^\circ = \dfrac{80 - x}{\sqrt{3}}$.

So $3x = 80 - x$, $x = 20$: **20 m from P, 60 m from Q**, and $h = 20\sqrt{3} \approx$ **34.64 m**.

Check: $\dfrac{34.64}{60} \approx 0.577 = \tan 30^\circ$ ✓.

MARKING SCHEME

- ✓ Both equations for h.
- ✓ $x = 20$, so 20 m and 60 m.
- ✓ $h = 20\sqrt{3} \approx 34.64$ m.

2 A flagpole on a building

ANSWER HA · 3

Distance to the building: $\tan 30^\circ = \dfrac{10}{d}$, so $d = 10\sqrt{3}$ m.

Top of the pole: $\tan 45^\circ = \dfrac{10 + p}{d}$, so $10 + p = 10\sqrt{3}$.

Flagpole $p = 10\sqrt{3} - 10 = 10(\sqrt{3} - 1) \approx$ **7.32 m**.

MARKING SCHEME

- ✓ $d = 10\sqrt{3}$ m.
- ✓ $10 + p = 10\sqrt{3}$.
- ✓ $p \approx 7.32$ m.

3 Two cars from a tower

ANSWER A · 3

The angles of elevation from the cars are 60° and 45° (alternate angles).

Near car: $\dfrac{100}{\tan 60^\circ} = \dfrac{100}{\sqrt{3}} \approx 57.74$ m. Far car: $\dfrac{100}{\tan 45^\circ} = 100$ m.

Apart: $100 - 57.74 =$ **42.26 m**.

MARKING SCHEME

- ✓ Near car $\dfrac{100}{\sqrt{3}} \approx 57.74$ m.
- ✓ Far car 100 m.
- ✓ 42.26 m.

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4 Show that

ANSWER HA · 3

Let the nearer distance be x. Then $x = h\cot\beta$ and $x + d = h\cot\alpha$.

Subtracting: $d = h(\cot\alpha - \cot\beta)$, so $h = \dfrac{d}{\cot\alpha - \cot\beta}$.

Check: $\cot 30^\circ = \sqrt{3}$, $\cot 60^\circ = \dfrac{1}{\sqrt{3}}$: $h = \dfrac{20}{\sqrt{3} - \dfrac{1}{\sqrt{3}}} = \dfrac{20\sqrt{3}}{2} = 10\sqrt{3}$ ✓.

MARKING SCHEME

- ✓ $x = h\cot\beta$ and $x + d = h\cot\alpha$.
- ✓ Subtracting gives the formula.
- ✓ The check gives $10\sqrt{3}$.

5 Shadows at two times

ANSWER HA · 3

Shadows: $\dfrac{h}{\tan 45^\circ} = h$ and $\dfrac{h}{\tan 60^\circ} = \dfrac{h}{\sqrt{3}}$.

$h - \dfrac{h}{\sqrt{3}} = 10$, so $h = \dfrac{10\sqrt{3}}{\sqrt{3} - 1} = 5(3 + \sqrt{3}) \approx$ **23.66 m**.

MARKING SCHEME

- ✓ The two shadows, h and $\dfrac{h}{\sqrt{3}}$.
- ✓ The equation $h - \dfrac{h}{\sqrt{3}} = 10$.
- ✓ $h \approx 23.66$ m.

6 The ladder slips

ANSWER A · 3

At 60°: top $6\sin 60^\circ = 3\sqrt{3} \approx 5.20$ m up; foot $6\cos 60^\circ = 3$ m out.

At 30°: top $6\sin 30^\circ = 3$ m up; foot $6\cos 30^\circ = 3\sqrt{3} \approx 5.20$ m out.

The top slides down $3\sqrt{3} - 3 \approx$ **2.20 m**, and the foot moves out the same, **2.20 m**.

MARKING SCHEME

- ✓ The top's heights, 5.20 m and 3 m.
- ✓ The foot's distances, 3 m and 5.20 m.
- ✓ Both 2.20 m.

7 Is the report possible?

ANSWER HA · 2

No. At 60° from 20 m, the top would be $20\tan 60^\circ = 20\sqrt{3} \approx 34.64$ m above the eye, a tree of about 36 m. A 10 m tree is only 8.5 m above the eye, which from 20 m gives an angle well under 30° ($20\tan 30^\circ \approx 11.5$ m is already more than 8.5 m).

MARKING SCHEME

- ✓ 60° from 20 m would need about 34.6 m above the eye.
- ✓ The tree's 8.5 m gives less than 30°, so the report is wrong.

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8 Make your own

ANSWER HA · 2

For example: "From a point 60 m from the foot of a mobile tower, the angle of elevation of its top is 30°. How tall is the tower?" $60 \tan 30° = \frac{60}{\sqrt{3}} = 20\sqrt{3} \approx 34.64$ m. Any question with a 30° angle whose answer is $20\sqrt{3}$ m is right.

MARKING SCHEME

- ✓ A clear question in a real setting with a 30° angle.
- ✓ A correct solution giving $20\sqrt{3}$ m.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	6	16	**22**

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