Unit 15 · Trigonometry

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. line of sight → **d.** the straight line from the eye to the object looked at
2. angle of elevation → **a.** the angle between the horizontal and the line of sight to an object above the eye
3. angle of depression → **b.** the angle between the horizontal and the line of sight to an object below the eye
4. clinometer → **c.** an instrument that measures an angle of elevation or depression
5. horizontal line → **f.** a level line through the eye, at right angles to a plumb line

K · 2.5

Each carries 0.5 mark.

2 Complete the set squares

1. $\sqrt{2}$
2. 2
3. 60°

U · 1.5

Each carries 0.5 mark.

3 Multiple choice

i. **c.** $\sqrt{3}$ — K · 1

In half an equilateral triangle, the side opposite 60° is $\sqrt{3}$ and the side next to it is 1: $\tan 60° = \frac{\sqrt{3}}{1}$.

ii. **b.** 1 — K · 1

$\frac{1}{2} + \frac{1}{2} = 1$.

iii. **b.** 45° — U · 1

$\tan\theta = \frac{20}{20} = 1$, so $\theta = 45°$.

iv. **b.** 40° — U · 1

The horizontals at the two ends are parallel and the line of sight crosses them, so the two angles are alternate angles: equal.

v. **a.** 30° — A · 1

$\tan\theta = \frac{h}{\sqrt{3}\,h} = \frac{1}{\sqrt{3}}$, so $\theta = 30°$.

4 Very short answers

a. MARKING SCHEME — K · 1

✓ $\frac{\sqrt{3}}{2}$.

b. MARKING SCHEME — K · 1

✓ tan.

c. MARKING SCHEME — U · 1

✓ 7 m.

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5 Short answers

a. **ANSWER** U · 2

The 40 m is the part of the pole **above his eye**; the part below his eye, 1.5 m, must be **added**, not subtracted.

The pole is $40 + 1.5 =$ **41.5 m** tall.

MARKING SCHEME

- ✓ The eye height must be added, not subtracted.
- ✓ 41.5 m.

b. **MARKING SCHEME** U · 2

- ✓ The two horizontals are parallel, cut by the line of sight.
- ✓ The angles are alternate angles, so equal.

c. **MARKING SCHEME** A · 2

- ✓ $AD = 30 \tan 45° = 30$ m.
- ✓ 31.6 m.

6 Problems

a. **ANSWER** A · 3

Height h, string the hypotenuse: $\sin 60° = \dfrac{h}{100}$.

$h = 100 \times \dfrac{\sqrt{3}}{2} = 50\sqrt{3} \approx$ **86.60 m**.

MARKING SCHEME

- ✓ The figure with the string as hypotenuse and the angle at the ground.
- ✓ $\sin 60° = \dfrac{h}{100}$.
- ✓ $50\sqrt{3} \approx 86.60$ m.

b. **ANSWER** A · 3

The angle of elevation of the top from the boat is also 30° (alternate angles). With d the distance:

$\tan 30° = \dfrac{30}{d}$.

$d = 30\sqrt{3} \approx$ **51.96 m**.

MARKING SCHEME

- ✓ The 30° at the boat (alternate angles).
- ✓ $\tan 30° = \dfrac{30}{d}$.
- ✓ $30\sqrt{3} \approx 51.96$ m.

c. **ANSWER** A · 3

Broken part x m, standing part $(9 - x)$ m: $\sin 30° = \dfrac{9 - x}{x}$, so $x = 18 - 2x$, $x = 6$.

Standing part **3 m**; the top lands $6 \cos 30° = 3\sqrt{3} \approx$ **5.20 m** from the foot.

MARKING SCHEME

- ✓ $\sin 30° = \dfrac{9 - x}{x}$.
- ✓ Standing part 3 m.
- ✓ $3\sqrt{3} \approx 5.20$ m.

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d. ANSWER HA · 4

Let the height be h and the second distance x.

From the near point: $\tan 60^\circ = \dfrac{h}{x}$, so $h = \sqrt{3}\,x$.

From the far point: $\tan 30^\circ = \dfrac{h}{x+20}$, so $\sqrt{3}\,h = x + 20$.

So $3x = x + 20$, $x = 10$, and $h = 10\sqrt{3} \approx$ **17.32 m**.

MARKING SCHEME

- ✓ The figure with both angles and the 20 m.
- ✓ $h = \sqrt{3}\,x$.
- ✓ $\sqrt{3}\,h = x + 20$.
- ✓ $x = 10$, $h = 10\sqrt{3} \approx 17.32$ m.

7 Think

ANSWER HA · 2

It **grows**: the height above your eye stays the same while the distance shrinks, so tan θ grows. It reaches **90° only when you stand right under the top**, looking straight up: then the distance is 0 and there is no triangle. At any distance it is less than 90°.

MARKING SCHEME

- ✓ The angle increases as the distance decreases.
- ✓ 90° only directly below the top (distance 0).

8 Activity: Measure a tall thing

(not marked)

WHAT STUDENTS SHOULD FIND

Each height is distance × tan(angle) + eye height (for 30°, 45° or 60° use the table; for other angles a calculator or the table at the back of a maths book). From two distances the heights should agree within about a meter; they disagree more when you stand very close or very far, where a degree's slip in reading the thread changes the answer most. Standing where the angle is near 45° gives the most reliable height.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	6.5	8.5	12	6	**33**

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