Unit 14 · Probability

HARDER WORKSHEET · ANSWER KEY

Full marks 23

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 At least one

ANSWER HA · 3

"At least one six" is everything except "no six". No six on a throw: $\frac{5}{6}$; on all three (independent): $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$.

$P(\text{at least one six}) = 1 - \frac{125}{216} = \mathbf{91/216}$ (about 0.42).

MARKING SCHEME

- ✓ At least one is $1 - P(\text{no six})$.
- ✓ $P(\text{no six}) = 125/216$.
- ✓ 91/216.

2 Find the bag

ANSWER HA · 3

$\frac{3}{n+3} \times \frac{2}{n+2} = \frac{1}{7}$, so $(n+3)(n+2) = 42 = 7 \times 6$, and $n + 3 = 7$: $n = \mathbf{4}$.

Check: $\frac{3}{7} \times \frac{2}{6} = \frac{6}{42} = \frac{1}{7}$ ✓.

MARKING SCHEME

- ✓ The product $\frac{3}{n+3} \times \frac{2}{n+2}$ set equal to 1/7.
- ✓ $(n+3)(n+2) = 42$.
- ✓ $n = 4$, checked.

3 Is it possible?

ANSWER HA · 2

i. **No.** Mutually exclusive would give $P(A \cup B) = 0.6 + 0.5 = 1.1$, more than 1.

ii. **No.** Independent would give $P(A \cap B) = 0.6 \times 0.5 = 0.3$, not 0.4.

MARKING SCHEME

- ✓ i. No: the union would be 1.1, more than 1.
- ✓ ii. No: independence needs 0.3.

4 Three draws

ANSWER A · 4

i. $\frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{210}{720} = \mathbf{7/24}$.

ii. One faulty can come first, second or third: three paths. Faulty first: $\frac{3}{10} \times \frac{7}{9} \times \frac{6}{8} = \frac{126}{720}$; the other two orders have the same numbers in another order, so the same product.

$P = 3 \times \frac{126}{720} = \frac{378}{720} = \mathbf{21/40}$.

MARKING SCHEME

- ✓ i. The three products 7/10, 6/9, 5/8.
- ✓ i. 7/24.
- ✓ ii. Three paths, each 126/720.
- ✓ ii. 21/40.

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5 How many times more?

ANSWER HA · 2

With replacement: $\frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$. Without: $\frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$.

With replacement, $\frac{221}{169} = \frac{17}{13} \approx$ **1.3 times** as likely: putting the ace back leaves 4 aces for the second card instead of 3.

MARKING SCHEME

- ✓ 1/169 and 1/221.
- ✓ With replacement, 17/13 ≈ 1.3 times.

6 Two traffic lights

ANSWER A · 3

i. Green then red, or red then green: $0.4 \times 0.5 + 0.6 \times 0.5 = 0.2 + 0.3 =$ **0.5**.

ii. Both green: $0.4 \times 0.5 =$ **0.2**.

MARKING SCHEME

- ✓ i. The two paths, 0.2 and 0.3.
- ✓ i. 0.5.
- ✓ ii. 0.2.

7 Spot the error

ANSWER U · 2

The three outcomes are not equally likely: "one of each" happens in two ways, HT and TH, of the four equally likely HH, HT, TH, TT. So $P = \frac{2}{4} =$ **1/2**.

MARKING SCHEME

- ✓ The outcomes aren't equally likely: one of each is HT and TH.
- ✓ 1/2.

8 Either or both

ANSWER A · 2

Multiples of 2: 15; of 5: 6; of both (multiples of 10): 3. Not mutually exclusive, so $P = \frac{15}{30} + \frac{6}{30} - \frac{3}{30} = \frac{18}{30} =$ **3/5**.

MARKING SCHEME

- ✓ 15, 6 and 3 in both, the 3 subtracted.
- ✓ 3/5.

9 Make your own

ANSWER HA · 2

Both red with replacement is $P(R)^2 = \frac{4}{9}$, so $P(R) = \frac{2}{3}$: two of every three balls red. **2 red and 1 blue**, 4 red and 2 blue, or 6 red and 3 blue all work: e.g. $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.

MARKING SCHEME

- ✓ $P(R) = 2/3$, since $(2/3)^2 = 4/9$.
- ✓ A bag with red : total = 2 : 3 (2 red 1 blue, 4 red 2 blue or 6 red 3 blue).

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	2	9	12	**23**

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