Unit 14 · Probability

ANSWER KEY · FOR TEACHERS

Full marks 32.5

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 2.5

1. sample space → **e.** the set of all possible outcomes of an experiment
2. mutually exclusive events → **a.** events that cannot happen together
3. independent events → **f.** events where one happening doesn't change the chance of the other
4. dependent events → **b.** events where the first changes the chance of the second
5. tree diagram → **c.** branches showing every outcome, stage by stage, with its probability

Each carries 0.5 mark.

2 Complete the tree

U · 2

1. 1/4
2. 3/4
3. 2/4
4. $2/5 \times 3/4 = 3/10$

Each carries 0.5 mark.

3 Multiple choice

i. **b.** 1/3 — K · 1

5 and 6: 2 of the 6 outcomes, $\frac{2}{6} = \frac{1}{3}$.

ii. **b.** a red card and a spade — U · 1

Every spade is black, so a card can't be both red and a spade. The king of hearts, every king, and the red aces show the others can happen together.

iii. **b.** 0.15 — U · 1

For independent events $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.5 = 0.15$.

iv. **b.** 3/9 — A · 1

After a red ball, 3 red are left of 9 balls: $\frac{3}{9}$.

v. **c.** 3/4 — A · 1

Of HH, HT, TH, TT, three have a tail: $\frac{3}{4}$. Or $1 - P(HH) = 1 - \frac{1}{4}$.

4 Very short answers

a. MARKING SCHEME — K · 1

✓ $P(A \cup B) = P(A) + P(B)$.

b. MARKING SCHEME — U · 1

✓ 0.65.

c. MARKING SCHEME — U · 1

✓ 8.

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5 Short answers

a. ANSWER U · 2

A king and a heart are not mutually exclusive: the king of hearts is counted twice. So subtract it once:

$P(K \cup H) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \mathbf{4/13}$.

MARKING SCHEME

- ✓ The king of hearts counted twice (the events are not mutually exclusive).
- ✓ $16/52 = 4/13$.

b. MARKING SCHEME U · 2

- ✓ With replacement the bag is unchanged, so the second draw's chances are the same.
- ✓ Without replacement the bag changes with the first ball, so the chances depend on it.

c. MARKING SCHEME A · 2

- ✓ 6 multiples of 3 and 2 of 7, none in both.
- ✓ $8/20 = 2/5$.

6 Problems

a. ANSWER A · 4

i. First ball: W 5/8, B 3/8. After W: W 4/7, B 3/7. After B: W 5/7, B 2/7.

$P(WW) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$, $P(WB) = \frac{15}{56}$, $P(BW) = \frac{15}{56}$, $P(BB) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$. (They add up to 1.)

ii. $P(WW) + P(BB) = \frac{20}{56} + \frac{6}{56} = \frac{26}{56} = \mathbf{13/28}$.

MARKING SCHEME

- ✓ The first draw's branches, 5/8 and 3/8.
- ✓ The second draw's branches after each, 4/7, 3/7 and 5/7, 2/7.
- ✓ $P(WW) = 20/56$ and $P(BB) = 6/56$.
- ✓ 13/28.

b. ANSWER U · 2

Independent: $P = \frac{3}{6} \times \frac{1}{2} = \mathbf{1/4}$.

MARKING SCHEME

- ✓ $P(\text{even}) = 1/2$, $P(H) = 1/2$, multiplied.
- ✓ 1/4.

c. ANSWER A · 3

i. $0.8 \times 0.7 = \mathbf{0.56}$.

ii. $0.2 \times 0.3 = \mathbf{0.06}$.

iii. $1 - 0.06 = \mathbf{0.94}$.

MARKING SCHEME

- ✓ 0.56.
- ✓ 0.06 from the chances of failing, 0.2 and 0.3.
- ✓ 0.94.

d. ANSWER HA · 3

i. $\frac{4}{r+4} = \frac{1}{3}$, so $r + 4 = 12$: **8 red balls**.

ii. $\frac{4}{12} \times \frac{3}{11} = \frac{12}{132} = \mathbf{1/11}$.

MARKING SCHEME

- ✓ The equation $\frac{4}{r+4} = \frac{1}{3}$.
- ✓ 8 red balls.
- ✓ 1/11.

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7 Think

ANSWER HA · 2

No. The coin has no memory: each toss is independent, so tails is still 1/2. The share of heads settles near 1/2 over many tosses because the early run gets swamped by thousands of later ones, not because tails catch up.

MARKING SCHEME

- ✓ No: the tosses are independent, so the probability is still 1/2.
- ✓ The relative frequency settles over many trials, not by making up for a run.

8 Activity: Two coins, 40 times

(not marked)

WHAT STUDENTS SHOULD FIND

The tree gives 1/4, 1/2, 1/4 (one head comes two ways, HT and TH), so about 10, 20 and 10 of the 40 tosses. Forty tosses swing a few either way; putting the whole class's results together brings the shares closer.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4.5	12	11	5	**32.5**

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