Unit 12 · Circle

HARDER WORKSHEET · ANSWER KEY

Full marks 23

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Work backwards

ANSWER A · 3

The central angle is double the inscribed angle: $5x - 10 = 2(2x + 20) = 4x + 40$, so $x = \mathbf{50}$.
$\angle ACB = \mathbf{120}°$ and reflex $\angle AOB = \mathbf{240}°$; the other angle $AOB = 360° - 240° = \mathbf{120}°$. ✓ An obtuse inscribed angle always stands on the major arc.

MARKING SCHEME

- ✓ The equation $5x - 10 = 2(2x + 20)$.
- ✓ $x = 50$: $\angle ACB = 120°$, reflex $\angle AOB = 240°$.
- ✓ The other angle $AOB = 120°$.

2 Angles from arcs

ANSWER A · 3

The arcs make 360°: 2 + 3 + 4 = 9 parts of 40°, so arc $AB = 80°$, arc $BC = 120°$, arc $CA = 160°$.
Each angle of the triangle is half the arc opposite it: $\angle C = 40°$, $\angle A = 60°$, $\angle B = 80°$. ✓ 40 + 60 + 80 = 180.

MARKING SCHEME

- ✓ The arcs 80°, 120°, 160°.
- ✓ Each angle half the opposite arc.
- ✓ $\angle A = 60°$, $\angle B = 80°$, $\angle C = 40°$.

3 Is it possible?

ANSWER HA · 2

The first, **no**: the opposite angles are 100° and 70° (170°), 80° and 110° (190°), not 180°. The second, **yes**: 100° + 80° = 180° and 70° + 110° = 180°, so it is cyclic.

MARKING SCHEME

- ✓ The first is not cyclic: opposite pairs 170° and 190°.
- ✓ The second is cyclic: both pairs 180°.

4 Show it

a. **ANSWER** HA · 4

1. $\angle DAB + \angle ADC = 180°$: co-interior angles, $AB \parallel DC$.
2. $\angle DAB + \angle BCD = 180°$: opposite angles of a cyclic quadrilateral.
3. $\angle ADC = \angle BCD$: from 1 and 2.
4. In $\triangle ADC$ and $\triangle BCD$: $\angle ADC = \angle BCD$ (from 3), $\angle DAC = \angle DBC$ (inscribed angles on the same arc DC), DC common.
5. $\triangle ADC$ cong $\triangle BCD$ by A.A.S., so $AD = BC$.

MARKING SCHEME

- ✓ $\angle ADC = \angle BCD$, from the two angle facts.
- ✓ $\angle DAC = \angle DBC$ with its reason.
- ✓ The triangles congruent by A.A.S.
- ✓ $AD = BC$.

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b. **ANSWER** HA · 4

Join AD.

1. $\angle AEC = \angle EAD + \angle EDA = \angle BAD + \angle CDA$: an exterior angle of $\triangle AED$.
2. $\angle BAD \triangleq \frac{1}{2}$ arc BD and $\angle CDA \triangleq \frac{1}{2}$ arc AC: an inscribed angle is half its arc.
3. $\angle AEC \triangleq \frac{1}{2}$(arc AC + arc BD): from 1 and 2.

MARKING SCHEME

- ✓ AD joined and the exterior angle of $\triangle AED$.
- ✓ $\angle BAD \triangleq \frac{1}{2}$ arc BD.
- ✓ $\angle CDA \triangleq \frac{1}{2}$ arc AC.
- ✓ The conclusion.

5 In a real setting

ANSWER A · 3

The row must take up 90° at the camera, so the row is seen at a right angle: the camera is on the **semicircle with the row as its diameter** (the angle in a semicircle), on the yard's side.
The semicircle's radius is $8 \div 2 = 4$ m, so the farthest place, straight in front of the middle, is **4 m** from the row.

MARKING SCHEME

- ✓ The angle 90° means the angle in a semicircle.
- ✓ The places: the semicircle on the row as diameter.
- ✓ The greatest distance 4 m.

6 Make your own

ANSWER HA · 2

Draw a circle with center O. Mark A on it, and with a protractor draw $\angle AOB = 70°$ with B on the circle. Mark any point C on the major arc and join CA and CB: $\angle ACB = \frac{1}{2} \times 70° = 35°$.

MARKING SCHEME

- ✓ A central angle of 70° drawn.
- ✓ C on the other (major) arc, so $\angle ACB = 35°$.

7 Think

ANSWER HA · 2

Bigger. Produce AP to meet the circle at C. Then $\angle APB$ is an exterior angle of $\triangle PCB$, so it is $\angle ACB + \angle PBC$, more than $\angle ACB$, and every inscribed angle on that arc is equal to $\angle ACB$.

MARKING SCHEME

- ✓ Bigger.
- ✓ A reason: the exterior angle of a triangle with the inscribed angle as one interior angle.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	9	14	**23**

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