Unit 9 · Indices

HARDER WORKSHEET · ANSWER KEY

Full marks 25

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Two bases at once

ANSWER A · 3

$4 \times 2^x \times \frac{3^x}{3} = 48$, so $6^x = 36$: $x = \mathbf{2}$.

MARKING SCHEME

✓ $2^{x+2} = 4 \times 2^x$ and $3^{x-1} = \frac{3^x}{3}$.

✓ $6^x = 36$.

✓ $x = 2$.

2 An index with a fraction

ANSWER A · 3

$4^{x-2} = 2^{2x-4}$, so $\frac{6}{x} = 2x - 4$, $2x^2 - 4x - 6 = 0$, $x^2 - 2x - 3 = 0$, $(x - 3)(x + 1) = 0$.

$x = \mathbf{3}$ **or** $\mathbf{-1}$ (both allowed, since only $x = 0$ is not). Check $x = -1$: $2^{-6} = 4^{-3} = \frac{1}{64}$ ✓.

MARKING SCHEME

✓ Both sides in base 2: $\frac{6}{x} = 2x - 4$.

✓ $x^2 - 2x - 3 = 0$.

✓ $x = 3$ or -1.

3 Hidden quadratic

ANSWER A · 3

Let $2^x = a$: $a + \frac{8}{a} = 9$, $a^2 - 9a + 8 = 0$, $a = 1$ or 8.

$x = \mathbf{0}$ **or** $\mathbf{3}$.

MARKING SCHEME

✓ $a^2 - 9a + 8 = 0$.

✓ $a = 1$ or 8.

✓ $x = 0$ or 3.

4 Work backwards

ANSWER HA · 3

Put $x = 1$: $4 - 2k + 8 = 0$, $k = 6$.

With $2^x = a$: $a^2 - 6a + 8 = 0$, $a = 2$ or 4. $k = \mathbf{6}$**, the other root** $x = \mathbf{2}$.

MARKING SCHEME

✓ $4 - 2k + 8 = 0$.

✓ $k = 6$.

✓ The other root 2.

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5 A real setting

ANSWER A · 3

After n doublings: $500 \times 2^n = 32000$, $2^n = 64 = 2^6$, $n = 6$: **2 hours**.
40 minutes before is $n = -2$: $500 \times 2^{-2} =$ **125 bacteria**.

MARKING SCHEME

- ✓ $500 \times 2^n = 32000$, $n = 6$.
- ✓ 2 hours.
- ✓ $500 \times 2^{-2} = 125$.

6 Spot the error

ANSWER U · 3

Powers are not added by adding their indices: $3^{x+1} + 3^x = 3 \times 3^x + 3^x = 4 \times 3^x$. And 36 is not 3^{12}: the bases must be equal before indices are compared.
$4 \times 3^x = 36$, $3^x = 9$: $x =$ **2**.

MARKING SCHEME

- ✓ $3^{x+1} + 3^x = 4 \times 3^x$, not 3^{2x+1}.
- ✓ Indices compared only with equal bases.
- ✓ $x = 2$.

7 Is the claim true?

ANSWER HA · 2

Let $5^x = a > 0$: $a + \frac{1}{a} = 1$ gives $a^2 - a + 1 = 0$, with $b^2 - 4ac = -3 < 0$: no real a. (Also $a + \frac{1}{a} \geq 2$ for any positive a.) **He is wrong: there is no solution.**

MARKING SCHEME

- ✓ $a^2 - a + 1 = 0$.
- ✓ No real root, so no solution.

8 Show it

ANSWER HA · 3

With $p = 2^{1/3}$, $q = 2^{-1/3}$, $pq = 1$: $x^3 = 2 + \frac{1}{2} + 3x$.
Multiply by 2: $2x^3 = 5 + 6x$, so $\mathbf{2x^3 - 6x = 5}$.

MARKING SCHEME

- ✓ $(p+q)^3 = p^3 + q^3 + 3pq(p+q)$ with $pq = 1$.
- ✓ $x^3 = \frac{5}{2} + 3x$.
- ✓ $2x^3 - 6x = 5$.

9 Make your own

ANSWER HA · 2

$3^x = 3$ or 9, so $(3^x - 3)(3^x - 9) = 0$: $\mathbf{9^x - 12 \times 3^x + 27 = 0}$.
$x = 1$: $9 - 36 + 27 = 0$ ✓. $x = 2$: $81 - 108 + 27 = 0$ ✓.

MARKING SCHEME

- ✓ An equation with $3^x = 3$ and $3^x = 9$ as its roots.
- ✓ Both values checked.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	12	10	**25**

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