Unit 9 · Indices

ANSWER KEY · FOR TEACHERS

Full marks 33.5

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. index → **f.** the number that shows how many times the base is multiplied — K · 2.5
2. exponential equation → **a.** an equation with the unknown in the index
3. a^0 → **d.** 1, for any base $a > 0$
4. a^{-n} → **e.** $\frac{1}{a^n}$
5. $a^{1/n}$ → **b.** the nth root of a

Each carries 0.5 mark.

2 Multiple choice

i. **c.** $\frac{1}{16}$ — K · 1

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$: a negative index means one over, not a negative number.

ii. **b.** 9 — K · 1

$27^{1/3} = 3$, and $3^2 = 9$.

iii. **c.** 8×2^x — U · 1

$2^{x+3} = 2^x \times 2^3 = 8 \times 2^x$. Adding indices means multiplying powers.

iv. **c.** 2 — U · 1

$1 = 3^0$, so $x - 2 = 0$ and $x = 2$.

v. **a.** 2 only — A · 1

$2^x = 4$ gives $x = 2$. $2^x = -1$ is impossible, since a power of 2 is always positive, so that value is rejected.

3 Very short answers

a. MARKING SCHEME — K · 1

✓ The rule with the condition on a.

b. MARKING SCHEME — U · 1

✓ $x = -3$.

c. MARKING SCHEME — U · 1

✓ $x = \frac{3}{2}$.

4 Short answers

a. ANSWER — U · 2

The index counts how many 2s are multiplied, not a division: $16 = 2^4$.

$x = \mathbf{4}$.

MARKING SCHEME

✓ $16 = 2^4$, not 2×8.

✓ $x = 4$.

b. MARKING SCHEME — U · 2

✓ Each step down the index divides by 2.

✓ $2 \div 2 = 1$.

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c. **MARKING SCHEME** A · 2

✓ 3^x taken out: $2 \times 3^x = 54$.

✓ $x = 3$.

5 Problems

a. **ANSWER** A · 3

$2^x(2 + 4 + 8) = 448$, $2^x = 32$: $x = \mathbf{5}$.

MARKING SCHEME

✓ 2^x taken out: $2^x(2 + 4 + 8) = 448$.

✓ $2^x = 32$.

✓ $x = 5$.

b. **ANSWER** A · 3

Let $4^x = a$: $16a^2 - 257a + 16 = 0$, $(a - 16)(16a - 1) = 0$, $a = 16$ or $\frac{1}{16}$. $x = \mathbf{\pm 2}$.

MARKING SCHEME

✓ $4^x = a$ and $16a^2 - 257a + 16 = 0$.

✓ $a = 16$ or $\frac{1}{16}$.

✓ $x = \pm 2$.

c. **ANSWER** A · 3

Let $3^x = a$: $a^2 - 12a + 27 = 0$, $(a - 3)(a - 9) = 0$. $x = \mathbf{1 \text{ or } 2}$.

MARKING SCHEME

✓ $3^x = a$ and $a^2 - 12a + 27 = 0$.

✓ $a = 3$ or 9.

✓ $x = 1$ or 2.

d. **ANSWER** A · 3

Let $3^x = a$: $3a^2 - 10a + 3 = 0$, $(3a - 1)(a - 3) = 0$, $a = \frac{1}{3}$ or 3.

$x = \mathbf{-1 \text{ or } 1}$.

MARKING SCHEME

✓ $3a^2 - 10a + 3 = 0$.

✓ $a = \frac{1}{3}$ or 3.

✓ $x = -1$ or 1.

e. **ANSWER** HA · 3

$pq = 3^{1/3} \times 3^{-1/3} = 1$, so cubing: $x^3 = 3 + \frac{1}{3} + 3x$. Multiply by 3: $3x^3 = 10 + 9x$, so $\mathbf{3x(x^2 - 3) = 10}$.

MARKING SCHEME

✓ $pq = 1$, the identity for $(p + q)^3$.

✓ $x^3 = \frac{10}{3} + 3x$.

✓ $3x(x^2 - 3) = 10$.

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6 Think

ANSWER HA · 2

$\left(\frac{1}{2}\right)^n = \frac{1}{32} = 2^{-5}$, so $n = 5$ half-lives: $5 \times 5730 =$ **28,650 years**, about 29,000 years.

The tree died about 29,000 years ago, in the last Ice Age.

MARKING SCHEME

✓ $n = 5$ from $\left(\frac{1}{2}\right)^n = \frac{1}{32}$.

✓ 28,650 years.

7 Activity: Folding paper

(not marked)

WHAT STUDENTS SHOULD FIND

Layers 1, 2, 4, 8, 16, 32: 2^n after n folds. Each layer is 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ of the sheet: 2^{-n}. At 0 folds the sheet is one layer, $2^0 = 1$. A share of $\frac{1}{256} = 2^{-8}$ needs 8 folds, which most sheets can't take (they get too thick).

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5.5	8	15	5	**33.5**

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