Unit 8 · Algebraic Fraction

HARDER WORKSHEET · ANSWER KEY

Full marks 26

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Cancel first

ANSWER A · 3

$x^3 - 8 = (x-2)(x^2+2x+4)$ and $x^3 + 8 = (x+2)(x^2-2x+4)$, so each fraction reduces:
$(x-2) + (x+2) = \mathbf{2}x$.

MARKING SCHEME

- ✓ $x^3 - 8$ factorized and canceled.
- ✓ $x^3 + 8$ factorized and canceled.
- ✓ $2x$.

2 Telescoping

ANSWER HA · 3

Each fraction splits: $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$. So the sum is
$\frac{1}{x} - \frac{1}{x+1} + \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+2} - \frac{1}{x+3} = \frac{1}{x} - \frac{1}{x+3} = \mathbf{\frac{3}{x(x+3)}}$.

MARKING SCHEME

- ✓ Each fraction split into two.
- ✓ The middle terms cancel.
- ✓ $\frac{3}{x(x+3)}$.

3 Signs turned round

ANSWER A · 3

Turn the denominators round so all use $(a-b)$, $(b-c)$, $(a-c)$: the LCM is $(a-b)(b-c)(a-c)$.
The numerator: $a(b-c) - b(a-c) + c(a-b) = ab - ac - ab + bc + ac - bc = 0$. **0.**

MARKING SCHEME

- ✓ Denominators turned round with their signs.
- ✓ The numerator $a(b-c) - b(a-c) + c(a-b)$.
- ✓ 0.

4 Work backwards

ANSWER A · 3

$p(x-2) + q(x+3) = 5x$: $(p+q)x + (3q-2p) = 5x$.
$p + q = 5$ and $3q = 2p$: $p = 3$, $q = 2$. $\mathbf{p = 3, q = 2}$.

MARKING SCHEME

- ✓ $p(x-2) + q(x+3) = 5x$.
- ✓ $p + q = 5$ and $3q - 2p = 0$.
- ✓ $p = 3$, $q = 2$.

5 A real setting

ANSWER

For a road of d km the time is $\frac{d}{u} + \frac{d}{v} = \frac{d(u+v)}{uv}$.

Average $= 2d \div \frac{d(u+v)}{uv} = \frac{2uv}{u+v}$.

$\frac{2 \times 30 \times 50}{80} = \mathbf{37.5\ km/h}$. The d canceled, so the length of the road doesn't matter.

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MARKING SCHEME HA · 4

- ✓ The total time $\frac{d}{u} + \frac{d}{v}$.
- ✓ $\frac{2uv}{u+v}$ shown.
- ✓ 37.5 km/h.
- ✓ The length canceled, so it doesn't matter.

6 Spot the error

ANSWER U · 3

The second numerator must be squared and in brackets: $(x+1)^2 - (x-1)^2$.

$= x^2 + 2x + 1 - x^2 + 2x - 1 = 4x$, so the answer is $\mathbf{\frac{4x}{x^2 - 1}}$.

MARKING SCHEME

- ✓ $(x-1)^2$ needed, in brackets.
- ✓ The numerator $4x$.
- ✓ $\frac{4x}{x^2-1}$.

7 Is the claim true?

ANSWER HA · 2

At $x = 1$: $\frac{7}{4} \neq 2$, so he is wrong.

$x + 6 = 2(x + 3)$ gives $x = 0$: only at $x = 0$ is it 2.

MARKING SCHEME

- ✓ A value with a different result, such as $x = 1$.
- ✓ Only $x = 0$.

8 Show it

ANSWER HA · 3

$a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 - a + 1)$, the LCM.

The first two: $\frac{(a^2 - a + 1) - (a^2 + a + 1)}{a^4 + a^2 + 1} = \frac{-2a}{a^4 + a^2 + 1}$.

Adding $\frac{2a}{a^4 + a^2 + 1}$ gives **0**.

MARKING SCHEME

- ✓ $a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 - a + 1)$.
- ✓ The first two make $\frac{-2a}{a^4 + a^2 + 1}$.
- ✓ The sum is 0.

9 Make your own

ANSWER HA · 2

For example $\frac{1}{x-1} + \frac{1}{x+1} = \frac{(x+1) + (x-1)}{x^2 - 1} = \frac{2x}{x^2 - 1}$.

Any pair whose sum simplifies to $\frac{2x}{x^2 - 1}$ is right.

MARKING SCHEME

- ✓ Two fractions with different denominators.
- ✓ Their sum shown to be $\frac{2x}{x^2 - 1}$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	9	14	**26**

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