Unit 8 · Algebraic Fraction

ANSWER KEY · FOR TEACHERS

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. algebraic fraction → **c.** a polynomial divided by a polynomial
2. lowest terms → **a.** numerator and denominator have no common factor
3. LCM of the denominators → **f.** each factor taken the most times it appears in any one denominator
4. like fractions → **b.** fractions with the same denominator
5. $b - a$ → **d.** $-(a - b)$

K · 2.5

Each carries 0.5 mark.

2 Fill the diagram

1. $x^2 - 4$
2. $x^2 + 4x + 4$
3. $2x - 4$

U · 1.5

Each carries 0.5 mark.

3 Multiple choice

i. **c.** 3 — K · 1

A fraction has no value when its denominator is 0: $x - 3 = 0$ at $x = 3$. At $x = -1$ the fraction is just 0.

ii. **a.** $x - 3$ — U · 1

$x^2 - 9 = (x + 3)(x - 3)$, and the factor $x + 3$ cancels.

iii. **b.** $(a - 1)(a + 1)^2$ — U · 1

$a^2 - 1 = (a - 1)(a + 1)$ and $a^2 + 2a + 1 = (a + 1)^2$: take $(a - 1)$ once and $(a + 1)$ twice. The plain product also works but has a factor too many.

iv. **a.** $\dfrac{2}{x - 2}$ — U · 1

$\dfrac{1}{2 - x} = -\dfrac{1}{x - 2}$, so the sum is $\dfrac{3 - 1}{x - 2}$. Adding 3 and 1 forgets the sign that turning $2 - x$ round brings.

v. **b.** $\dfrac{d}{24}$ — A · 1

$\dfrac{d}{40} + \dfrac{d}{60} = \dfrac{3d + 2d}{120} = \dfrac{5d}{120} = \dfrac{d}{24}$. Its average speed is $2d \div \dfrac{d}{24} = 48$ km/h, not 50.

4 Very short answers

a. MARKING SCHEME — K · 1

✓ Both factors correct.

b. MARKING SCHEME — U · 1

✓ 1.

c. MARKING SCHEME — U · 1

✓ $\dfrac{a}{a + 4}$.

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5 Short answers

a. ANSWER U · 2

At $x = 2$: $\frac{8}{4} = 2$, but $x + 2 = 4$. **He is wrong.**

He canceled terms, not factors: $x^2 + 4$ doesn't factorize, so nothing cancels.

MARKING SCHEME

- ✓ The values 2 and 4 differ.
- ✓ Only factors cancel; $x^2 + 4$ has no factor $x + 2$.

b. MARKING SCHEME U · 2

- ✓ The LCM $x^2 - y^2$ and the numerator with brackets.
- ✓ $\frac{2y}{x^2 - y^2}$.

c. MARKING SCHEME A · 2

- ✓ $a(x + 1) + (x - 1) = 3x + 1$.
- ✓ $a = 2$.

6 Problems

a. ANSWER U · 2

$$\frac{(a+b)^2 + (a-b)^2}{a^2 - b^2} = \frac{\mathbf{2(a^2 + b^2)}}{\mathbf{a^2 - b^2}}$$

MARKING SCHEME

- ✓ The numerator $(a + b)^2 + (a - b)^2$ over $a^2 - b^2$.
- ✓ $\frac{2(a^2 + b^2)}{a^2 - b^2}$.

b. ANSWER A · 3

The denominators are $(x - 2)(x - 3)$, $(x - 1)(x - 2)$, $(x - 1)(x - 3)$; the LCM is $(x - 1)(x - 2)(x - 3)$.
The numerator: $(x - 1) + (x - 3) - 2(x - 2) = 0$. **0.**

MARKING SCHEME

- ✓ The three denominators factorized.
- ✓ The LCM and the numerator $(x - 1) + (x - 3) - 2(x - 2)$.
- ✓ 0.

c. ANSWER A · 3

$4x - x^2 - 3 = -(x - 1)(x - 3)$; the LCM is $(x - 1)(x - 2)(x - 3)$. The numerator:
$(x - 1) - 2(x - 2) - 3(x - 3) = -4x + 12 = -4(x - 3)$.
$-\frac{\mathbf{4}}{\mathbf{(x-1)(x-2)}}$, the same as $\frac{4}{3x - x^2 - 2}$.

MARKING SCHEME

- ✓ Each denominator factorized, $4x - x^2 - 3$ turned round with its sign.
- ✓ The LCM $(x - 1)(x - 2)(x - 3)$ and the numerator $-4x + 12$.
- ✓ $\frac{4}{3x - x^2 - 2}$ (or $-\frac{4}{(x-1)(x-2)}$).

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d. **ANSWER** A · 4

Two at a time: $\frac{1}{a-1} - \frac{1}{a+1} = \frac{2}{a^2-1}$.

$\frac{2}{a^2-1} - \frac{2}{a^2+1} = \frac{4}{a^4-1}$.

$\frac{4}{a^4-1} - \frac{4}{a^4+1} = \frac{\mathbf{8}}{a^{\mathbf{8}}-\mathbf{1}}$.

MARKING SCHEME

✓ $\frac{2}{a^2-1}$.

✓ $\frac{4}{a^4-1}$.

✓ The last step set up over $a^8 - 1$.

✓ $\frac{8}{a^8-1}$.

7 Think

ANSWER HA · 2

For a road of d km: time $\frac{d}{4} + \frac{d}{12} = \frac{d}{3}$ hours, so the average is $2d \div \frac{d}{3} = \mathbf{6\ km/h}$.

She spends three times as long walking as running, so the slow speed counts for more than the fast one.

MARKING SCHEME

✓ 6 km/h.

✓ More time is spent at the slow speed.

8 Activity: Check it with numbers

(not marked)

WHAT STUDENTS SHOULD FIND

The two sides are equal for every x tried: $\frac{x(x+2)-2(x-2)}{x^2-4} = \frac{x^2+4}{x^2-4}$. For example $x = 3$: $3 - \frac{2}{5} = \frac{13}{5}$ on both sides; $x = 0$: -1 on both. At $x = 2$ (and -2) neither side has a value, because a denominator is 0.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4.5	12.5	13	2	**32**

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