Unit 7 · Quadratic Equation

HARDER WORKSHEET · ANSWER KEY

Full marks 29

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Work backwards

ANSWER A · 3

Put $x = 3$: $9 - 3k + 12 = 0$, so $k = 7$.
$x^2 - 7x + 12 = 0$, $(x - 3)(x - 4) = 0$: **$k = 7$, and the other root is 4.**

MARKING SCHEME

- ✓ $9 - 3k + 12 = 0$.
- ✓ $k = 7$.
- ✓ The other root 4.

2 Two equal roots

ANSWER HA · 3

Equal roots need $b^2 - 4ac = 0$: $k^2 - 64 = 0$, so $k = 8$ or -8.
$k = 8$: $(x + 4)^2 = 0$, root -4. $k = -8$: $(x - 4)^2 = 0$, root 4.
$k = \pm 8$; the root is -4 or 4.

MARKING SCHEME

- ✓ $k^2 - 64 = 0$.
- ✓ $k = 8$ or -8.
- ✓ The roots -4 and 4.

3 Fractions that hide a quadratic

ANSWER A · 3

Multiply by $6x(x + 1)$: $6x^2 + 6(x + 1)^2 = 13x(x + 1)$, so $12x^2 + 12x + 6 = 13x^2 + 13x$.
$x^2 + x - 6 = 0$, $(x + 3)(x - 2) = 0$: **$x = 2$ or -3**.
Check $x = 2$: $\frac{2}{3} + \frac{3}{2} = \frac{13}{6}$ ✓.

MARKING SCHEME

- ✓ Fractions cleared correctly.
- ✓ $x^2 + x - 6 = 0$.
- ✓ $x = 2$ or -3.

4 A real setting

ANSWER A · 3

Path width x m: the grass is $(16 - 2x)$ m by $(10 - 2x)$ m.
$(16 - 2x)(10 - 2x) = 72$, $4x^2 - 52x + 88 = 0$, $x^2 - 13x + 22 = 0$, $(x - 2)(x - 11) = 0$.
A path 11 m wide doesn't fit in a 10 m garden, so **the path is 2 m wide** (grass 12 m × 6 m = 72 m² ✓).

MARKING SCHEME

- ✓ $(16 - 2x)(10 - 2x) = 72$.
- ✓ $x^2 - 13x + 22 = 0$, $x = 2$ or 11.
- ✓ 2 m, with 11 rejected and why.

5 Two taps

ANSWER

Smaller tap x hours, bigger $x - 5$ hours. In one hour they fill $\frac{1}{x}$ and $\frac{1}{x - 5}$ of the tank:

$\frac{1}{x} + \frac{1}{x - 5} = \frac{1}{6}$, so $6(x - 5) + 6x = x(x - 5)$, $x^2 - 17x + 30 = 0$, $(x - 15)(x - 2) = 0$.
$x = 2$ would give the bigger tap -3 hours, so **the smaller tap takes 15 hours and the bigger 10**

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hours. HA · 4

Check: $\frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{1}{6}$ ✓.

MARKING SCHEME

✓ $\frac{1}{x} + \frac{1}{x-5} = \frac{1}{6}$.

✓ $x^2 - 17x + 30 = 0$.

✓ $x = 15$ (2 rejected).

✓ 15 hours and 10 hours.

6 Spot the error

ANSWER U · 3

Two sign mistakes: $-b = -(-3) = 3$, and $-4ac = -4 \times 2 \times (-5) = +40$.

$x = \frac{3 \pm \sqrt{9+40}}{4} = \frac{3 \pm 7}{4}$: $x = \mathbf{\frac{5}{2}}$ **or −1**.

Check: $2(-1)^2 - 3(-1) - 5 = 2 + 3 - 5 = 0$ ✓.

MARKING SCHEME

✓ $-b$ is 3, not -3.

✓ $b^2 - 4ac = 49$, not $9 - 40$.

✓ $x = \frac{5}{2}$ or -1.

7 Is the claim true?

ANSWER HA · 3

Length x, breadth y: $x + y = 20$ and $xy = 120$, so $x(20 - x) = 120$, $x^2 - 20x + 120 = 0$.

$b^2 - 4ac = 400 - 480 = -80$, negative: no real length works. **She can't be right.**

With a perimeter of 40 m, the largest area is a 10 m × 10 m square, 100 m^2.

MARKING SCHEME

✓ $x^2 - 20x + 120 = 0$.

✓ $b^2 - 4ac = -80 < 0$.

✓ Not possible (no real roots).

8 Show it

ANSWER HA · 2

$x^2 + 2x + 1 + 4 = 0$, so $(x + 1)^2 = -4$.

The square of a real number is never negative, so no real x makes $(x + 1)^2 = -4$: **no real root**.

MARKING SCHEME

✓ $(x + 1)^2 = -4$.

✓ A square can't be negative, so no real root.

9 Numbers and digits

ANSWER A · 3

Tens digit x, units digit $x + 3$: the number is $10x + (x + 3) = 11x + 3$.

$x(x + 3) = 11x + 3 - 15$, so $x^2 - 8x + 12 = 0$, $(x - 2)(x - 6) = 0$.

Both roots are digits: **25 and 69**. Check: $2 \times 5 = 10 = 25 - 15$ ✓, $6 \times 9 = 54 = 69 - 15$ ✓.

MARKING SCHEME

✓ $x(x + 3) = 11x + 3 - 15$.

✓ $x = 2$ or 6.

✓ Both numbers, 25 and 69.

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10 Make your own

ANSWER HA · 2

For example: "My two cousins' ages differ by 1 year, and their product is 56. How old are they?"
$x(x + 1) = 56$, $(x + 8)(x - 7) = 0$: $x = 7$ (an age can't be -8), so **7 and 8 years**.
Any story whose equation is $x^2 + x - 56 = 0$ is right: a rectangle of area 56 m^2 whose length is 1 m more than its breadth (7 m by 8 m) works too.

MARKING SCHEME

- ✓ A story that leads to $x^2 + x - 56 = 0$.
- ✓ Solved, with -8 rejected and the story's answer given.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	12	14	**29**

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