# Unit 7 · Quadratic Equation

**ANSWER KEY · FOR TEACHERS**

Full marks 36.5

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

## 1 Key words

1. quadratic equation → **d.** an equation in one variable whose highest power is 2
2. standard form → **g.** $ax^2 + bx + c = 0$, with $a \neq 0$
3. roots → **f.** the values of the variable that make the equation true
4. factorization method → **b.** writing one side as a product of two factors, with 0 on the other side
5. completing the square → **a.** adding the square of half the coefficient of $x$ to both sides
6. $b^2 - 4ac$ → **e.** the number under the square root that decides how many real roots there are

Each carries 0.5 mark. **K · 3**

## 2 Fill the diagram

1. x m
2. 300 $m^2$
3. (2x + 1) m

Each carries 0.5 mark. **U · 1.5**

## 3 Multiple choice

**i.** **b.** $(x + 1)(x - 1) = x^2 + 2x$ **K · 1**

Multiplied out, $x^2 - 1 = x^2 + 2x$: the squares cancel and $2x = -1$ is linear.

**ii.** **a.** 4 and $-\frac{3}{2}$ **U · 1**

$x - 4 = 0$ gives 4; $2x + 3 = 0$ gives $-\frac{3}{2}$. The signs flip, and the 2 divides.

**iii.** **c.** 25 **U · 1**

Half of the coefficient $-10$ is $-5$, and $(-5)^2 = 25$: $x^2 - 10x + 25 = (x - 5)^2$.

**iv.** **b.** one (both roots equal) **U · 1**

$b^2 - 4ac = 144 - 144 = 0$. Indeed $4x^2 - 12x + 9 = (2x - 3)^2$, so only $x = \frac{3}{2}$.

**v.** **a.** $x^2 + 3x - 40 = 0$ **A · 1**

$x(x + 3) = 40$, so $x^2 + 3x - 40 = 0$ (and $x = 5$). $2x + 3$ is the sum of the two sides, not the area.

## 4 Very short answers

**a.** **MARKING SCHEME** **K · 1**

✓ The formula written correctly.

**b.** **MARKING SCHEME** **K · 1**

✓ The standard form with $a$, $b$, $c$ and their signs.

**c.** **MARKING SCHEME** **U · 1**

✓ Both 4 and −4.



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## 5 Short answers

**a.** MARKING SCHEME U · 2

- ✓ Only a product of 0 tells you a factor is 0.
- ✓ $x = 6$ or $-3$.

**b.** ANSWER U · 2

Dividing by $x$ throws away the root $x = 0$ (you can't divide by $x$ if it might be 0).
$x^2 - 5x = 0$, $x(x - 5) = 0$: $x =$ **0 or 5**.

MARKING SCHEME

- ✓ Dividing by $x$ loses the root 0.
- ✓ $x = 0$ or 5.

**c.** MARKING SCHEME A · 2

- ✓ $(x - 3)(x + 5) = 0$.
- ✓ $x^2 + 2x - 15 = 0$.

## 6 Problems

**a.** ANSWER U · 3

Divide by 2: $x^2 + 4x = 5$. Add $2^2 = 4$: $(x + 2)^2 = 9$, so $x + 2 = \pm 3$.
$x =$ **1 or −5.**

MARKING SCHEME

- ✓ $x^2 + 4x = 5$ and 4 added to both sides.
- ✓ $(x + 2)^2 = 9$, $x + 2 = \pm 3$.
- ✓ $x = 1$ or $-5$.

**b.** ANSWER A · 3

$a = 3$, $b = -4$, $c = -2$: $b^2 - 4ac = 16 + 24 = 40$.
$x = \dfrac{4 \pm \sqrt{40}}{6} = \dfrac{4 \pm 6.325}{6}$: $x \approx$ **1.72 or −0.39**.

MARKING SCHEME

- ✓ $a$, $b$, $c$ and $b^2 - 4ac = 40$.
- ✓ $x = \dfrac{4 \pm \sqrt{40}}{6}$.
- ✓ 1.72 and −0.39.

**c.** ANSWER A · 3

$x + y = 25$, $xy = 150$: $x^2 - 25x + 150 = 0$, $(x - 15)(x - 10) = 0$.
**Length 15 m, breadth 10 m.**

MARKING SCHEME

- ✓ $x + y = 25$ and $xy = 150$.
- ✓ $x^2 - 25x + 150 = 0$.
- ✓ Length 15 m, breadth 10 m.

**d.** ANSWER A · 3

$(34 + x)(4 + x) = 400$, $x^2 + 38x - 264 = 0$, $(x + 44)(x - 6) = 0$.
**In 6 years** ($40 \times 10 = 400$ ✓).

MARKING SCHEME

- ✓ $(34 + x)(4 + x) = 400$.
- ✓ $x^2 + 38x - 264 = 0$, $(x + 44)(x - 6) = 0$.
- ✓ In 6 years (−44 rejected).

**e.** ANSWER

a. $\dfrac{180}{x - 5} - \dfrac{180}{x} = 3$. Dividing by 3 and multiplying by $x(x - 5)$: $60x - 60(x - 5) = x(x - 5)$, so
$x^2 - 5x - 300 = 0$, $(x - 20)(x + 15) = 0$. **20 students.**


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b. 15 were present: $180 \div 15 = \mathbf{12\ pencils\ each}$ (3 more than the planned 9). **A · 4**

**MARKING SCHEME**

- ✓ $\dfrac{180}{x-5} - \dfrac{180}{x} = 3$.
- ✓ $x^2 - 5x - 300 = 0$.
- ✓ 20 students ($-15$ rejected).
- ✓ 12 pencils each.

## 7 Think

**ANSWER** **HA · 2**

$20t - 5t^2 = 15$, so $t^2 - 4t + 3 = 0$, $(t-1)(t-3) = 0$: **after 1 s and after 3 s**.
Both make sense: the ball passes 15 m once on the way up and once on the way down.

**MARKING SCHEME**

- ✓ $t = 1$ and $t = 3$.
- ✓ Once going up, once coming down, so both are kept.

## 8 Activity: One string, many rectangles

(not marked)

**WHAT STUDENTS SHOULD FIND**

The length is $12 - x$: 9, 7 and 6 cm, and the areas 27, 35 and 36 cm$^2$. An area of 32 cm$^2$ needs $x(12 - x) = 32$, $x^2 - 12x + 32 = 0$, $x = 4$ or 8: the rectangle 4 cm by 8 cm (the two roots are the same rectangle turned round). An area of 40 cm$^2$ is impossible: $x^2 - 12x + 40 = 0$ has $b^2 - 4ac = 144 - 160 < 0$. The largest area is the square, 6 cm by 6 cm, 36 cm$^2$.

| Marks by level | Knowledge | Understanding | Application | Higher ability | Total |
|---|---|---|---|---|---|
| **Marks** | 6 | 12.5 | 16 | 2 | **36.5** |


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