Unit 6 · Sequence and Series

HARDER WORKSHEET · ANSWER KEY

Full marks 23

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Work backwards

ANSWER HA · 3

$t_1 = S_1 = 5$. $S_2 = 16$, so $t_2 = 16 - 5 = 11$. $S_3 = 33$, so $t_3 = 33 - 16 = 17$.
5, 11, 17 go up by 6. In general $t_n = S_n - S_{n-1} = 3n^2 + 2n - 3(n-1)^2 - 2(n-1) = 6n - 1$, which goes up by **6** every time: **an arithmetic series, $a = 5$, $d = 6$.**

MARKING SCHEME

- ✓ $t_1 = 5$ and $t_2 = 11$.
- ✓ $t_3 = 17$.
- ✓ $t_n = 6n - 1$ (or the difference is the same, 6): arithmetic.

2 A real setting

ANSWER A · 3

The interest goes 1000, 900, 800, …, 100: an arithmetic series with $a = 1000$, $d = -100$, $n = 10$, $l = 100$.
$S_{10} = \frac{10}{2}(1000 + 100) =$ **Rs 5,500**.

MARKING SCHEME

- ✓ The interest payments make an arithmetic series, 1000 down to 100.
- ✓ $n = 10$, $l = 100$.
- ✓ Rs 5,500.

3 Two means, two numbers

ANSWER HA · 3

$\frac{a+b}{2} = 25$, so $a + b = 50$; $\sqrt{ab} = 7$, so $ab = 49$.
Then $a(50 - a) = 49$, $a^2 - 50a + 49 = 0$, $(a - 1)(a - 49) = 0$.
The numbers are 1 and 49. Check: average 25, $\sqrt{49} = 7$. ✓

MARKING SCHEME

- ✓ $a + b = 50$ and $ab = 49$.
- ✓ $a^2 - 50a + 49 = 0$.
- ✓ 1 and 49.

4 Show it

ANSWER HA · 2

$S_5 = \frac{5}{2}[2a + 4d] = 5(a + 2d)$, and $a + 2d$ is the third, middle term. So $5t_3 = 100$ and $t_3 = 20$, whatever a and d are.
For example 16, 18, 20, 22, 24 and 0, 10, 20, 30, 40.

MARKING SCHEME

- ✓ $S_5 = 5(a + 2d) = 5t_3$, so $t_3 = 20$.
- ✓ Two correct sequences with middle term 20.

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5 Spot the error

ANSWER U · 3

The bottom of the formula is $1 - r$, not $1 + r$: $1 - \frac{1}{4} = \frac{3}{4}$.

$$S_5 = \frac{64 \times \frac{1023}{1024}}{\frac{3}{4}} = \frac{1023}{12} = \mathbf{85.25}.$$

Check: $64 + 16 + 4 + 1 + 0.25 = 85.25$. ✓

MARKING SCHEME

- ✓ The mistake: the denominator is $1 - r$.
- ✓ The correct working with $\frac{3}{4}$.
- ✓ 85.25.

6 When does it overtake?

ANSWER HA · 3

$S_A = \frac{n}{2}[4 + 3(n-1)] = \frac{n(3n+1)}{2}$; $S_B = 2^n - 1$.

$n = 5$: A 40, B 31. $n = 6$: A 57, B 63.

After 6 terms. A adds 3 more each time; B doubles, and doubling always wins in the end.

MARKING SCHEME

- ✓ Both sums written as formulas in n.
- ✓ The values at $n = 5$ and $n = 6$.
- ✓ 6 terms.

7 Is the claim true?

ANSWER HA · 3

After n days the total is $\frac{1(2^n - 1)}{2 - 1} = 2^n - 1$.

$2^{19} - 1 = 524287$, $2^{20} - 1 = 1048575$. **True: the total passes Rs 10 lakh on the 20th day**, inside three weeks.

But on that day alone you must save 2^{19} = Rs 5,24,288: the plan is true on paper and impossible in practice.

MARKING SCHEME

- ✓ The total after n days is $2^n - 1$.
- ✓ Day 19: 5,24,287; day 20: 10,48,575.
- ✓ True, on the 20th day.

8 A bouncing ball

ANSWER A · 3

Bounces: 4 m, 3.2 m, 2.56 m before the fourth hit, each counted up and down.

Distance $= 5 + 2(4 + 3.2 + 2.56) = 5 + 19.52 =$ **24.52 m**.

For ever: $5 + 2 \times \frac{4}{1 - \frac{4}{5}} = 5 + 40 =$ **45 m**, never passed.

MARKING SCHEME

- ✓ The three bounces 4, 3.2 and 2.56 m, counted twice.
- ✓ 24.52 m.
- ✓ Never more than 45 m.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	6	14	**23**

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