Unit 6 · Sequence and Series

ANSWER KEY · FOR TEACHERS

Full marks 34

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. arithmetic means → **f.** the terms between two terms of an arithmetic sequence
2. geometric means → **e.** the terms between two terms of a geometric sequence
3. common difference → **h.** the number added to each term of an arithmetic sequence
4. common ratio → **b.** the number each term of a geometric sequence is multiplied by
5. series → **a.** the terms of a sequence written with plus signs between them
6. $\frac{n}{2}(a + l)$ → **c.** the sum of an arithmetic series
7. $\frac{a(r^n - 1)}{r - 1}$ → **d.** the sum of a geometric series

K · 3.5

Each carries 0.5 mark.

2 Fill the diagram

1. 8
2. 13
3. 18

U · 1.5

Each carries 0.5 mark.

3 Multiple choice

i. **b.** $\sqrt{ab}$ — K · 1

If a, m, b is geometric, $m^2 = ab$. $\frac{a + b}{2}$ is the arithmetic mean.

ii. **c.** 7 — U · 1

The 5 means and the 2 ends: $n = m + 2 = 7$.

iii. **b.** 420 — U · 1

20 terms, average of first and last = 21: $20 \times 21 = 420$. 840 forgets to halve.

iv. **c.** 189 — A · 1

$\frac{3(2^6 - 1)}{2 - 1} = 3 \times 63 = 189$. 96 is the 6th term; 192 forgets the "− 1".

v. **c.** 16 — A · 1

$\frac{n}{2}[60 - 4(n - 1)] = 0$ gives $64 - 4n = 0$, $n = 16$: the terms 30, 26, …, −30 pair off to zero.

4 Very short answers

a. MARKING SCHEME — K · 1

✓ The formula written correctly.

b. MARKING SCHEME — U · 1

✓ 7.

c. MARKING SCHEME — U · 1

✓ 12.

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5 Short answers

a. MARKING SCHEME U · 2

- ✓ Each pair (column) adds to 51.
- ✓ $50 \times 51 \div 2 = 1275$.

b. MARKING SCHEME U · 2

- ✓ The number of terms is 41.
- ✓ 1,640.

c. MARKING SCHEME A · 2

- ✓ $r^3 = 27$, $r = 3$.
- ✓ 12 and 36.

6 Problems

a. ANSWER A · 3

$a + 2d = -15$ and $a + 7d = 10$: $5d = 25$, $d = 5$, $a = -25$.
$S_{16} = 8[-50 + 15 \times 5] = 8 \times 25 = \mathbf{200}$.

MARKING SCHEME

- ✓ $a + 2d = -15$ and $a + 7d = 10$.
- ✓ $d = 5$, $a = -25$.
- ✓ $S_{16} = 200$.

b. ANSWER A · 3

$S_{10} = 5[2000 + 9 \times 100] = 5 \times 2900 = 14500$.
No: he makes 14,500, 500 short of 15,000.

MARKING SCHEME

- ✓ $a = 1000$, $d = 100$, $n = 10$.
- ✓ $S_{10} = 14500$.
- ✓ No, 500 short.

c. ANSWER A · 3

$ar = 4$ and $ar^6 = 128$: dividing, $r^5 = 32$, $r = 2$, $a = 2$.
$S_{10} = 2(2^{10} - 1) = \mathbf{2{,}046}$.

MARKING SCHEME

- ✓ $r^5 = 32$, so $r = 2$.
- ✓ $a = 2$.
- ✓ $S_{10} = 2046$.

d. ANSWER A · 4

$43680 = \dfrac{a(3^6 - 1)}{2} = 364a$, so $a = 120$.
Last: $ar^5 = 120 \times 243 = 29160$. Difference: $29160 - 120 = \mathbf{Rs\ 29{,}040}$.

MARKING SCHEME

- ✓ $43680 = \dfrac{a(3^6 - 1)}{3 - 1}$.
- ✓ $a = 120$.
- ✓ Last installment $120 \times 3^5 = 29160$.
- ✓ Difference Rs 29,040.

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7 Think

ANSWER HA · 2

Up to the fifth hit: $2 + 2(1 + 0.5 + 0.25 + 0.125) = 2 + 3.75 = 5.75$ m. Each bounce adds half as much as the one before, so the bracket never reaches 2, and the whole distance stays under $2 + 2 \times 2 = 6$ m.
He is not right.

MARKING SCHEME

- ✓ 5.75 m up to the fifth hit.
- ✓ The total stays under 6 m, because each bounce adds half the one before.

8 Activity: Measure a bounce

(not marked)

WHAT STUDENTS SHOULD FIND

The ratios come out roughly equal (a tennis ball about 0.5–0.6, a good rubber ball higher), so the heights make a geometric sequence. With a ratio r, the distance up to the fourth hit is $100 + 2(100r + 100r^2 + 100r^3)$ cm: for $r = 0.55$, about $100 + 2(55 + 30.3 + 16.6) \approx 304$ cm.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5.5	9.5	17	2	**34**

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