Unit 3 · Growth and Depreciation

HARDER WORKSHEET · ANSWER KEY

Full marks 24

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Find the unknown

ANSWER **A · 3**

From year 2 to year 4 the population is multiplied by $(1 + R/100)^2 = 14641/12100 = 1.21 = 1.1^2$, so $R =$ **10%**.

Start: $P \times 1.1^2 = 12100$, so $P = 12100/1.21 =$ **10,000**.

MARKING SCHEME

- ✓ $(1 + R/100)^2 = 14641/12100 = 1.21$.
- ✓ $R = 10\%$.
- ✓ Start 10,000.

2 When does it overtake?

ANSWER **HA · 3**

A needs $1.04^T > 60000/50000 = 1.2$.

$1.04^4 \approx 1.1699$ (58,493 people, still fewer); $1.04^5 \approx 1.2167$ (60,833 people).

After 5 years.

MARKING SCHEME

- ✓ The condition $1.04^T > 1.2$.
- ✓ After 4 years about 58,493; after 5 about 60,833.
- ✓ 5 years.

3 Is the claim true?

ANSWER **HA · 3**

After 5 years it keeps $0.9^5 \approx 0.59$ of its value: **not half, about 59%**. He has added $5 \times 10\%$ as if every year took 10% of the first price.

$0.9^6 \approx 0.53$, $0.9^7 \approx 0.48$. It is first worth less than half **after 7 years**.

MARKING SCHEME

- ✓ $0.9^5 \approx 0.59$: not right.
- ✓ $0.9^6 \approx 0.53$ and $0.9^7 \approx 0.48$.
- ✓ 7 years.

4 Spot the error

MARKING SCHEME **U · 3**

- ✓ The mistake: the newcomers grow only after they arrive.
- ✓ 23,000 at the end of year 1.
- ✓ 24,150.

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5 Show it

ANSWER HA · 3

$P(1 + R/100)(1 - R/100) = P(1 - R^2/100^2) = P(1 - R^2/10000)$ (difference of two squares). Since $R^2/10000 > 0$, the bracket is less than 1.
For 30%: $1000(1 - 900/10000) = 910$: **Rs 90 lower**.

MARKING SCHEME

- ✓ The product written as $P(1 + R/100)(1 - R/100)$.
- ✓ $= P(1 - R^2/10000)$, less than P.
- ✓ Rs 910, Rs 90 lower.

6 Work backwards

ANSWER A · 3

Before the second year's growth: $25410/1.05 = 24200$. Before the 1,000 left: $24200 + 1000 = 25200$.
At the start: $25200/1.05 =$ **24,000**.
Check: $24000 \times 1.05 = 25200$, $25200 - 1000 = 24200$, $24200 \times 1.05 = 25410$ ✓.

MARKING SCHEME

- ✓ 24,200 at the start of year 2.
- ✓ 25,200 before the people left.
- ✓ 24,000 at the start.

7 In a real setting

ANSWER A · 4

Pay: $20000 \times 1.05^2 = 22050$, up by a factor of 1.1025.
Prices: $1.0774 \times 1.0544 \approx 1.1360$.
$1.1025/1.1360 \approx 0.9705$. Her pay buys **less, about 3% less** than at the start, although it rose by 10.25%.

MARKING SCHEME

- ✓ Pay factor 1.1025 (Rs 22,050).
- ✓ Price factor about 1.1360.
- ✓ Divides one by the other: about 0.97.
- ✓ Less, by about 3%.

8 Make your own

ANSWER HA · 2

The increase is $P[(1 + R/100)^2 - 1]$. For example $R = 10$: $0.21P = 2100$, so $P = 10000$. Check: $10000 \to 11000 \to 12100$, 2,100 more ✓. (Another that works: 1,680 at 50%: $1680 \to 2520 \to 3780$, 2,100 more. Many choices give a fraction of a person, so the check matters.)

MARKING SCHEME

- ✓ A population and rate with $P[(1 + R/100)^2 - 1] = 2100$.
- ✓ A check that it grows by 2,100.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	10	11	**24**

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