Unit 1 · Sets

HARDER WORKSHEET · ANSWER KEY

Full marks 30

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Find the unknown

a. **ANSWER** **A · 3**

$3x + (2x + 5) - x = 45$, so $4x + 5 = 45$ and $x = 10$.
$n(B) = 2 \times 10 + 5 = 25$, so $n_0(B) = 25 - 10 = 15$.

MARKING SCHEME

- ✓ $3x + 2x + 5 - x = 45$ written.
- ✓ $x = 10$.
- ✓ $n_0(B) = 25 - 10 = 15$.

b. **ANSWER** **A · 4**

Let both be k: tea only $3k$, coffee only $2k$. Then $3k + 2k + k = 6k$ is everyone who likes at least one, which is $100\% - 12\% = 88\%$.
$k = 44$, so $6k = 264$ people are 88% of the total, and the total is $264 \div 0.88 = 300$.

i. **300 people.**
ii. Tea: $3 \times 44 + 44 =$ **176**.

MARKING SCHEME

- ✓ The three regions inside the circles are 6 parts, and they are 88% of the total.
- ✓ $6 \times 44 = 264$.
- ✓ Total: $264 \div 0.88 = 300$.
- ✓ Tea: $132 + 44 = 176$.

2 The least and the most

ANSWER **HA · 4**

i. The union can't be more than 1,200, so both $\geq 900 + 800 - 1200 = 500$.
ii. The overlap can't be more than the smaller set: 800 (every smartphone household also has a mobile phone).
iii. At least one: $1200 - 150 = 1050$. Both: $900 + 800 - 1050 = 650$.
iv. $800 - 650 = 150$.

MARKING SCHEME

- ✓ i. 500 (from $900 + 800 - 1200$).
- ✓ ii. 800, the smaller set.
- ✓ iii. 650 (from at least one = 1050).
- ✓ iv. 150.

3 Is it possible?

ANSWER **HA · 3**

At least one game: $60 + 50 + 40 - 20 - 15 - 25 + 5 = 95$. Then $95 + 10 = 105$, more than the 100 students asked, so the numbers can't all be right.
If only "none" is wrong, it should be $100 - 95 = 5$.

MARKING SCHEME

- ✓ $n(F \cup C \cup V) = 95$.
- ✓ $95 + 10 = 105 > 100$, so the report can't be right.
- ✓ None should be 5.

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4 Spot the error

MARKING SCHEME **U · 3**

- ✓ The pairs' sum, 31, is right.
- ✓ The mistake: each pair includes the 6 in all three clubs, so they are counted three times.
- ✓ Exactly two: $31 - 18 = 13$.

5 Show it

ANSWER **HA · 3**

The union is three regions: $n(A \cup B) = n_0(A) + n(A \cap B) + n_0(B)$. Take $n(A \cap B)$ from both sides: $n_0(A) + n_0(B) = n(A \cup B) - n(A \cap B)$.
Exactly one: $64 - 14 =$ **50**.

MARKING SCHEME

- ✓ $n(A \cup B) = n_0(A) + n(A \cap B) + n_0(B)$ written (from the diagram).
- ✓ Rearranged to $n_0(A) + n_0(B) = n(A \cup B) - n(A \cap B)$.
- ✓ 50.

6 A count with a trick

ANSWER **HA · 4**

$50 + 45 + 40 = 135$ counts for 100 people, so there are 35 extra counts. A person who likes exactly two fruits is counted once extra; one who likes all three, twice extra.
Let each "two only" region hold x: $3x + 2 \times 4 = 35$, so $3x = 27$ and $x =$ **9**.
Exactly one: $100 - 3 \times 9 - 4 =$ **69**.
(Check: apples only $50 - 9 - 9 - 4 = 28$, bananas only $45 - 22 = 23$, guavas only $40 - 22 = 18$, and $28 + 23 + 18 = 69$. ✓)

MARKING SCHEME

- ✓ $135 - 100 = 35$ extra counts.
- ✓ $3x + 2 \times 4 = 35$ (one extra for each in exactly two, two extra for each in all three).
- ✓ $x = 9$.
- ✓ Exactly one: $100 - 27 - 4 = 69$.

7 In a real setting

ANSWER **A · 4**

Let A, B, D be the donors with each marker. $n(A \cup B \cup D) = 72 + 58 + 180 - 14 - 52 - 64 + 12 = 192$.
i. O−: $200 - 192 =$ **8**.
Pairs only: A and B $14 - 12 = 2$, B and D $52 - 12 = 40$, A and D $64 - 12 = 52$.
ii. O+: $180 - (12 + 40 + 52) =$ **76**.
A−: $72 - (12 + 2 + 52) = 6$.
iii. O− + O+ + A− + A+: $8 + 76 + 6 + 52 =$ **142 donors**.

MARKING SCHEME

- ✓ $n(A \cup B \cup D) = 192$, so O− is 8.
- ✓ O+: $180 - 104 = 76$.
- ✓ A− is 6 and A+ is 52 (the "A and D only" region).
- ✓ $8 + 76 + 6 + 52 = 142$.

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8 Make your own

ANSWER HA · 2

For example, $n_0(A) = 20$ and $n_0(B) = 10$: then $n(A) = 32$, $n(B) = 22$, and neither is $50 - 20 - 12 - 10 = 8$. Check: $32 + 22 - 12 + 8 = 50$. ✓ Any two "only" numbers that add up to 30 work, and neither is always 8.

MARKING SCHEME

- ✓ "Only" regions adding up to 30 with both 12, and $n(A)$, $n(B)$ found from them.
- ✓ Neither = 8, with a check that the regions add up to 50.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	3	11	16	**30**

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