Unit 1 · Sets

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. universal set → **e.** the set of everything being talked about
2. cardinal number → **c.** how many elements a set has
3. disjoint sets → **g.** sets with no element in common
4. intersection → **a.** the elements in both sets
5. $n_0(A)$ → **f.** the number of elements in A only
6. $n(\overline{A \cup B})$ → **b.** the number of elements in neither A nor B

Each carries 0.5 mark. **K · 3**

2 Fill the diagram

1. 9
2. 6
3. 4
4. 5

Each carries 0.5 mark. **U · 2**

3 Multiple choice

i. **b.** $n(B)$ **K · 1**

Every element of A is already in B, so joining them adds nothing new: $n(A \cup B) = n(B)$.

ii. **b.** everywhere except the overlap **U · 1**

"Not both" leaves out only those who like both: the people who like one game, and those who like neither. "Neither" is only the region outside both circles.

iii. **c.** 9 **U · 1**

$P \cap Q$ is the "P and Q only" region together with the very middle, so the "only" part is $15 - 6 = 9$.

iv. **c.** 5 **A · 1**

At least one: $30 + 25 - 10 = 45$, so neither: $50 - 45 = 5$.

v. **c.** 20% **A · 1**

At least one is 100%, so both is $70\% + 50\% - 100\% = 20\%$. (120% is the two added together, which counts the 20% twice.)

4 Very short answers

a. **MARKING SCHEME** **K · 1**

✓ $n(\overline{M \cup S})$ written (or $n(U) - n(M \cup S)$).

b. **MARKING SCHEME** **A · 1**

✓ 25.

c. **MARKING SCHEME** **K · 1**

✓ 8.

5 Short answers

a. **MARKING SCHEME** **U · 2**

✓ Adding $n(A)$ and $n(B)$ counts the elements in both sets twice.

✓ Taking $n(A \cap B)$ away once leaves each element counted once.

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b. **MARKING SCHEME** U · 2

✓ The overlap is already counted in both 25 and 18, so it is subtracted, not added.

✓ 36.

c. **MARKING SCHEME** A · 2

✓ $n(A \cap B) = 12$.

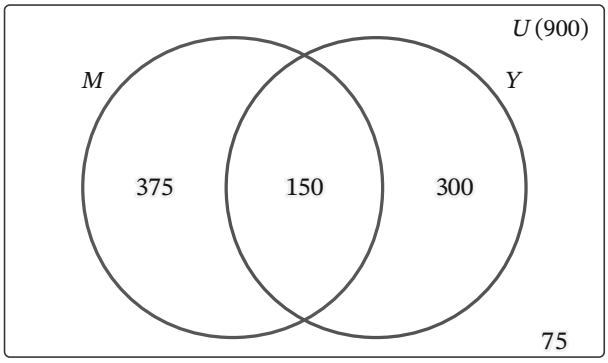
✓ $n(A) = 30$.

6 Survey problems

a. **ANSWER** A · 4

Let M and Y be the readers of *Madhupark* and *Yuba Manch*. At least one: $900 - 75 = 825$.

ii. Both: $525 + 450 - 825 = \mathbf{150}$.



iii. Only one: $375 + 300 = \mathbf{675}$.

MARKING SCHEME

✓ A Venn diagram with all four regions and the rectangle U.

✓ At least one: $900 - 75 = 825$.

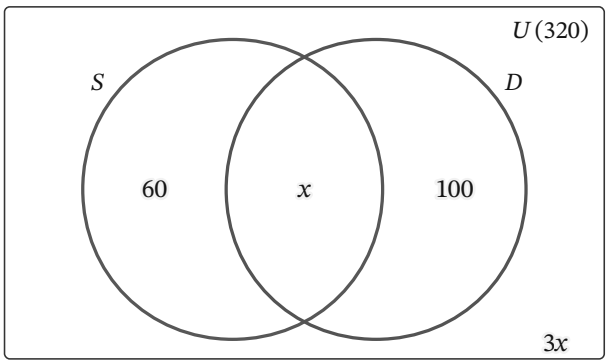
✓ Both: $525 + 450 - 825 = 150$.

✓ Only one: $375 + 300 = 675$.

b. **ANSWER** A · 4

Let S and D be the participants who sing and dance.

Let $n(S \cap D) = x$, so neither is $3x$.



$60 + x + 100 + 3x = 320$, so $4x = 160$ and $x = 40$.

ii. Neither: $3 \times 40 = \mathbf{120}$.

iii. At most one (everyone except those who do both): $320 - 40 = \mathbf{280}$.

MARKING SCHEME

✓ A Venn diagram with both as x and neither as $3x$.

✓ $60 + x + 100 + 3x = 320$, so $x = 40$.

✓ Neither: $3 \times 40 = 120$.

✓ At most one: $320 - 40 = 280$.

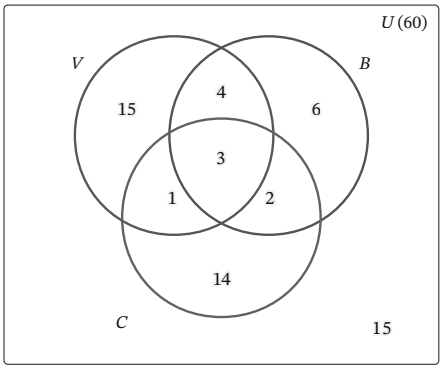
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c. **ANSWER** **A · 4**

Let V, B and C be the students who played volleyball, basketball and cricket, and $n(V \cap B \cap C) = x$.

ii. $n(V \cup B \cup C) = 60 - 15 = 45$. So $45 = 23 + 15 + 20 - 7 - 5 - 4 + x = 42 + x$, and $x = \mathbf{3}$.

iii. Volleyball and cricket only: $4 - 3 = \mathbf{1}$.



MARKING SCHEME

- ✓ $n(V \cup B \cup C) = 60 - 15 = 45$.
- ✓ $45 = 23 + 15 + 20 - 7 - 5 - 4 + x$, so all three: $x = 3$.
- ✓ Volleyball and cricket only: $4 - 3 = 1$.
- ✓ A Venn diagram with all eight regions (15, 6, 14, 4, 2, 1, 3 and 15 outside).

7 Think

ANSWER **HA · 2**

Yes. At most 40 students like at least one, so $n(F \cap C) = 25 + 22 - n(F \cup C) \geq 47 - 40 = 7$. The overlap is largest when every cricket lover also likes football ($C \subset F$): **22**.

MARKING SCHEME

- ✓ Hari is right: the union can't be more than 40, so both $\geq 25 + 22 - 40 = 7$.
- ✓ The most is 22, when all who like cricket also like football.

8 Activity: Survey your class

(not marked)

WHAT STUDENTS SHOULD FIND

The four regions (cricket only, both, football only, neither) add up to the number of students asked, and $n(C) + n(F)$ is more than the number who like at least one game by exactly the number who like both. If the check fails, someone was counted twice or missed.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	6	8	17	2	**33**

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